**Team Epsilon**

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Proposition 4.8

Proposition 4.8: Hilbert’s parallel postulate ⇔ converse to Theorem 4.1 (alternate interior angles).

(⇒) Assume Hilbert’s parallel postulate.

Suppose l is parallel to m, and t is a transversal to l and m, intersecting l at P and m at Q. Let A and B be two points on l on the opposite sides of t. Let C be a point on m on the same side of t as A. Suppose ∠CQP is not congruent to ∠BPQ (RAA). By C-4, there is a unique ray QD on the same side of t as C such that ∠DQP is congruent to ∠BPQ. By Theorem 4.1 (AIA) QD is parallel to l. But this contradicts Hilbert’s parallel postulate since m and QD are two lines through Q parallel to l. Hence the converse to Theorem 4.1 holds.

(⇐) Assume the converse of Theorem 4.1. Suppose l is any line and Q is any point not on l. As in

Corollary 2 to Theorem 4.1, there exists a unique line t through Q perpendicular to l. Also, there exists a

unique line m through Q perpendicular to t. Then Corollary 1 gives that l is parallel to m. Suppose n is a line through Q parallel to l. Let P be the point common to t and l, and let A not equal to P be any point on l. Let B and C be points on m and n, respectively, on the opposite side of t as A. Since l and m are parallel ∠APQ is congruent to ∠BQP by the converse to Theorem 4.1. Similarly, ∠APQ is congruent to ∠CQP, so that ∠CQP ∼= ∠BQP. By the uniqueness of C-4, ray QB is congruent to ray QC. Hence, m = n, and Hilbert’s parallel postulate holds.