4.1. We start with $Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$.

An eigenvector with $\lambda = 1$ would be $\begin{bmatrix} -i \\ 1 \end{bmatrix}$.

We would like to see this on the Bloch sphere.

There are two problems, the vector is not normalized, nor does it have a positive real first component.

We can divide by $-i$, which solves the problem of having a real first component $\begin{bmatrix} 1 \\ i \end{bmatrix}$. (This is allowed since if $\lambda v = Av$ then $\lambda i v = A(\lambda i v)$, so $iv$ is again an eigenvector.)

We still need to normalize the length, since this vector has length squared $\begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix} \begin{bmatrix} 1 \\ i \end{bmatrix} = 1 - i^2 = 2$.

Hence what we need is $\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \end{bmatrix}$.

This corresponds to $\theta = \pi$, $\cos \frac{\theta}{2} = \frac{\sqrt{2}}{2}$, $\sin \frac{\theta}{2} = \frac{\sqrt{2}}{2}$

and $e^{i\Psi} = i$, so $\Psi = \pi/2$. 