We solve the initial value problem using `dsolve`, which solves differential equations. However, getting a formula solution for this first order linear equation is only part of the job.

\[
\begin{align*}
&\text{de:=diff}(p(t),t)=1/2*p(t)-450; \\
&\text{inits:=p}(0)=850; \\
&\text{sol:=dsolve}([\text{de,inits}],p(t)); \\
&\text{de} := \frac{d}{dt} p(t) = \frac{1}{2} p(t) - 450 \\
&\text{inits} := p(0) = 850 \\
&\text{sol} := p(t) = 900 - 50 e^{\frac{1}{2} t}
\end{align*}
\]  

(1)

a) To find out the time of extinction, we set the population at time t equal to zero, using the `rhs` (right hand side) command so that we do not need to retype the formula for the population at time t.

Once the equation is created, we solve for t. (`dsolve` solve differential equations, while `solve` works with equations that are not differential.)

The float of the exact answer is 5.78 months.

\[
\begin{align*}
&\text{eq:=0}=\text{rhs}(\text{sol}); \\
&\text{solve(eq,\{t\});} \\
&\text{evalf(\%);}
\end{align*}
\]

\[
\begin{align*}
eq := 0 &= 900 - 50 e^{\frac{1}{2} t} \\
\{t=2 \ln(18)\} \\
\{t=5.780743516\}
\end{align*}
\]  

(2)

b) Note that the critical point is 900. Above that initial population, the population does not become extinct. That is the reason why the problem asks you to assume that \( p_0 < 900 \).

\[
\begin{align*}
&\text{eq:=0}=\text{rhs}(\text{de}); \\
&\text{solve(\%,p(t));}
\end{align*}
\]

\[
\begin{align*}
eq &= 0 = \frac{1}{2} p(t) - 450 \\
900
\end{align*}
\]  

(3)

We can again use `dsolve`.

\[
\begin{align*}
&\text{de;} \\
&\text{inits:=p}(0)=p[0]; \\
&\text{sol:=dsolve}([\text{de,inits}],p(t)); \\
&\frac{d}{dt} p(t) = \frac{1}{2} p(t) - 450
\end{align*}
\]
The solution for $t$ gives the number of months before extinction.

\[
inits := p(0) = p_0
\]
\[
sol := p(t) = 900 + e^{\frac{t}{2}} \left( p_0 - 900 \right)
\]  \hspace{1cm} (4)

The solution for $t$ gives the number of months before extinction.

\[
eq := 0 = rhs(sol);
\]
\[
tsolve := t = solve(%, t);
\]
\[
\text{eq} := 0 = 900 + e^\frac{t}{2} \left( \frac{p_0 - 900}{p_0 - 900} \right)
\]
\[
tsolve := t = 2 \ln \left( -\frac{900}{p_0 - 900} \right)
\]  \hspace{1cm} (5)

Setting the time to extinction to 12 months, we then solve for $p_0$. In this situation, we would like a floating point decimal as a solution, so we use evaluate as a float, or evalf.

\[
eq := 12 = rhs(tsolve);
\]
\[
solve(\%, p[0]);
\]
\[
evalf(\%);
\]
\[
eq := 12 = 2 \ln \left( -\frac{900}{p_0 - 900} \right)
\]
\[
\frac{900 \left( e^6 - 1 \right)}{e^6}
\]
\[
897.7691230
\]  \hspace{1cm} (6)

We solve the initial value problem.

\[
de := \text{diff}(v(t), t) = \frac{98}{10} - \frac{28}{100} v(t);
\]
\[
inits := v(0) = 0;
\]
\[
sol := \text{dsolve}\{\text{de}, \text{inits}\}, v(t)\};
\]
\[
de := \frac{dv}{dt} v(t) = \frac{49}{5} - \frac{7}{25} v(t)
\]
\[
inits := v(0) = 0
\]
\[
sol := v(t) = 35 - 35 e^{-\frac{7}{25} t}
\]  \hspace{1cm} (7)

a) As $t$ becomes large, the velocity approaches 35. Hence, we can form an equation that sets the velocity at time $t$ equal to $98/100$ of the terminal velocity, to which we can then apply Maple’s solve command.

\[
eq := 98/100 * 35 = rhs(sol);
\]  \hspace{1cm} (8)
The hailstone takes about 13.97 second to fall that fast.

\[
eq := \frac{343}{10} = 35 - 35 e^{-\frac{7}{25} t}
\]  \hspace{1cm} (8)

The distance fallen is the definite integral of the velocity over the time interval. In particular, the hailstone falls 366.5 meters.

> t=solve(%,t);
evalf(%);

\[
t = \frac{25}{7} \ln(50)
\]

\[
t = 13.97151073
\]  \hspace{1cm} (9)

> int(rhs(sol),t=0..rhs(%) );

\[
366.5028756
\]  \hspace{1cm} (10)