Verify explicit solutions.

\[ \begin{align*}
\text{de} & := 2t^2 \frac{\partial^2 y(t)}{\partial t^2} + 3t \frac{\partial y(t)}{\partial t} - y(t) = 0; \\
\text{sol} & := y(t) = c_1 t^{1/2} + c_2 t^{-1}; \\
\end{align*} \]

Since the verification is true regardless of the choices of \(c_1\) and \(c_2\), then it must be true for \(c_1 = 1\) and \(c_2 = 0\), hence just the first solution, as well as for \(x_1 = 0\) and \(c_2 = 1\), i.e., just the second solution. One line of Maple code suffices to check both.

\[ \begin{align*}
> \text{subs(sol, de);} \\
> \text{simplify(\%);} \\
2t^2 \left( \frac{\partial^2}{\partial t^2} \left( c_1 \sqrt{t} + \frac{c_2}{t} \right) \right) + 3t \left( \frac{\partial}{\partial t} \left( c_1 \sqrt{t} + \frac{c_2}{t} \right) \right) - c_1 \sqrt{t} - \frac{c_2}{t} = 0 \\
0 = 0
\end{align*} \]

Verify an explicit solution.

\[ \begin{align*}
\text{de} & := t^2 \frac{\partial^2 y(t)}{\partial t^2} + 4t \frac{\partial y(t)}{\partial t} + 2y(t) = 0; \\
\text{sol} & := y(t) = t^r; \\
\end{align*} \]

\[ > \text{subs(sol, de);} \\
> \text{simplify(\%);} \\
> \text{\%/t^r;} \\
> \text{solve(\%,\{r\});} \\
\end{align*} \]

\[ \begin{align*}
> \text{\left( r^2 + 3r + 2 \right) = 0} \\
> r^2 + 3r + 2 = 0 \\
> \{ r = -1 \}, \{ r = -2 \}
\end{align*} \]