In the interval $(0,1)$, none of the Heavisides is awake, so we get zero. In the interval $(1,3)$, only the first Heaviside is awake, so we get $1$ times $1$, or just $1$. In the interval $(3,4)$, both the first and the second of the three Heavisides are awake, so we get $1$ times $1$ plus $2$ times $1$, or $3$, in that interval. After $t = 4$, all the Heaviside are awake and we get 

\[
Heaviside(t-1)+2\cdot Heaviside(t-3)-6\cdot Heaviside(t-4);
\]

\[
plot(%,t=-1..6,discont=true);
\]

\[
Heaviside(t-1) + 2\cdot Heaviside(t-3) - 6\cdot Heaviside(t-4)
\]

We can convert the Heavisides into a piecewise function, by hand.

Since the Laplace integral starts at $t = 0$, we need only consider values of $t > 0$. In the interval $(0,2)$, both Heavisides are asleep, so the value of the function is $(t - 3) \cdot 0 - (t - 2) \cdot 0 = 0$. In the interval $(2, 3)$, only the first Heaviside is awake, so the value of the function is $(t - 3) \cdot 1 - (t - 2) \cdot 0 = t - 3$. In the interval $(3, \infty)$, both Heavisides are awake, so the value of the function is $(t - 3) \cdot 1 - (t - 2) \cdot 1 = -1$. 

Alternatively, we can ask Maple to convert:

\[
(t - 3) \text{Heaviside}(t - 2) - (t - 2) \text{Heaviside}(t - 3);
\]

\[
\text{convert(}, \%, \text{piecewise};
\]

\[
(t - 3) \text{Heaviside}(t - 2) - (t - 2) \text{Heaviside}(t - 3)
\]

\[
\begin{align*}
0 & \quad t < 2 \\
\text{undefined} & \quad t = 2 \\
(t - 3) & \quad t < 3 \\
\text{undefined} & \quad t = 3 \\
-1 & \quad 3 < t
\end{align*}
\]

(1)

One of the ways to compute the Laplace transform is to break the Laplace integral into three pieces, as was done on p. 315, #14. Note that the last integral is an improper integral, and we need to choose \( s > 0 \) to make \( -\frac{1}{e^{st}} \) go quickly to zero, so that the integral converges.

\[
\text{restart:}
\]
\[
\text{assume(s>0):}
\]
\[
\text{Int}(0*\exp(-s*t),t=0..2)+\text{Int}((t-3)*\exp(-s*t),t=2..3)+\text{Int}(-1*\exp(-s*t),t=3..\text{infinity});
\]
\[
\text{value(});
\]
\[
\text{value(});
\]
\[
\text{value(});
\]
\[
\text{value(});
\]
\[
\text{simplify(});
\]

\[
\begin{align*}
\int_0^2 0 \, dt + \int_2^3 (t - 3) e^{-st} \, dt + \int_3^\infty (-e^{-st}) \, dt \\
&= -\frac{e^{2s}s - e^{2s} + e^{3s}}{s^2} + \frac{1}{s(e^s)^3} \\
&= -\frac{e^{2s}s - e^{2s} + e^{3s} + e^{3s}s}{s^2}
\end{align*}
\]

(2)

A second way to do the problem is to use Maple's laplace command in the inttrans package:

\[
\text{with(inttrans):}
\]
\[
(t-3)*\text{Heaviside}(t-2)-(t-2)*\text{Heaviside}(t-3);
\]
\[
\text{laplace(}, \%, t, s);
\]

\[
(t - 3) \text{Heaviside}(t - 2) - (t - 2) \text{Heaviside}(t - 3)
\]

\[
- \frac{(s + 1)e^{-3s} + (s - 1)e^{-2s}}{s^2}
\]

(3)

Still a third way to do the problem is to apply line 13 in Table 5.3.1, which requires us to express \((t - 3)\) in terms of powers of the \((t - 2)\) that occurs in \(\text{Heaviside}(t - 2)\), as \((t - 3) = (t - 2) - 1\). Hence the first
term in the problem becomes two entries in the table: \((t - 2)\) Heaviside\((t - 2)\) - 1 Heaviside\((t - 2)\). (Note that 1 is the zeroth power of \((t - 2)\).)

According to line 13, the Laplace of \((t - 2)\) Heaviside\((t - 2)\) is \(\frac{e^{-2s}}{s^2}\), with the \(f\) in the table being \(t\) and the \(F\) being \(\frac{1}{s^2}\). The Laplace of -1 Heaviside\((t - 2)\) is \(-\frac{e^{-2s}}{s}\), with the \(f\) in the table being -1 and the \(F\) being \(-\frac{1}{s}\).

Similarly, to apply line 13 in Table 5.3.1, we need to convert \(-(t - 2)\) in terms of powers of the \((t - 3)\) in Heaviside\((t - 3)\). In particular \(-\frac{3}{(s - 2)^4}\), which we recognize as \(\frac{3!}{(s - 2)^4}\), with \(s\) replaced by \((s - 2)\). Now by line 3 in the table, we see that \(\frac{3!}{(s - 2)^4}\) is the Laplace transform of \(t^3\), and the replacement of \(s\) by \((s - 2)\) is due to multiplication by \(e^{2t}\). Hence, the inverse Laplace transform of \(\frac{3!}{(s - 2)^4}\) is \(e^{2t}t^3\).

Now, when we come back from Laplace, we always multiply the \(f(t)\) from the table by Heaviside\((t)\). Hence we get \(e^{2t}t^3\) Heaviside\((t)\). The final step is to take account of the \(e^{-s}\) that we covered up at the beginning. The effect of multiplication by \(e^{-s}\) is to replace each \(t\) by \((t - 1)\), including the \(t\) in the Heaviside\((t)\).
The final answer is then $e^{2(t-1)}(t-1)^3$ Heaviside$(t-1)$.

\[
> \text{with(inttrans):} \\
3!*\exp(-s)/(s-2)^4; \\
\text{invlaplace(%,s,t);} \\
\frac{6\ e^{-s}}{(s-2)^4} \\
\text{Heaviside}(t-1)\ (t-1)^3\ e^{2(t-2)}
\]  

(p. 348 #15)

Cover up the exponential $e^{-2s}$. Now take the inverse Laplace transform of \(\frac{2(s-1)}{s^2 - 2s + 2}\), remembering to append the Heaviside(t).

We recognize that the discriminant of \(s^2 - 2s + 2\) is -4, so the denominator does not factor over the reals. Hence, we complete the square. Now \(\frac{2(s-1)}{s^2 - 2s + 2} = \frac{2(s-1)}{(s-1)^2 + 1}\), which is \(\frac{2 - s}{s^2 + 1}\) with the s replaced by \((s-1)\). Thus, the inverse Laplace transform is \(e^t 2 \cos(t)\) Heaviside$(t)$. The effect of the \(e^{-2s}\) that we ignored is to replace each \(t\) by \((t - 2)\).

Thus, the final answer is \(e^t 2 \cos(t - 2)\) Heaviside$(t - 2)$.

\[
> \text{with(inttrans):} \\
2*(s-1)*\exp(-2*s)/(s^2-2*s+2); \\
\text{invlaplace(%,s,t);} \\
\frac{2(s-1)e^{-2s}}{s^2-2s+2} \\
2 \text{Heaviside}(t-2)\ e^{t-2} \cos(t-2)
\]