MATH 409 Homework 1

1. For each of the following sets say whether they are bounded from above and/or below. Also, indicate the supremum and infimum if the set possesses one:
   a. $\{n \in \mathbb{Z} : \ cos(n!)\}$
   b. $\cap_{n=1}^{\infty}(\frac{1}{n}, 1 + \frac{1}{n})$.
2. Does the sequence $x_n = \sqrt{4n^2 + n + 2} - 2n$ have a limit; if so what is its value?
3. Prove that $\sqrt{2} + \sqrt{3}$ is irrational.
4. Prove that $\sqrt{n-1} + \sqrt{n+1}$ is irrational for every positive integer n.
5. For the following statements, if true give a proof, if false provide a counterexample:
   a. $\{x_n\}$ is a sequence of rational numbers such that x_n converges to x, $x_n \to x$. Then the number x must be rational.
   b. $\{y_n\}$ is a sequence of irrational numbers (that is, $y_n \in \mathbb{R}, y_n \notin \mathbb{Q}$) is such that y_n converges to y. Then the number y must be irrational.