

MATH 409 Homework 5

1. We know that $f(x) = x^3$ is uniformly continuous on $I = [-2, 3]$ (why)?
For any $\epsilon > 0$, find a formula for $\delta = \delta(\epsilon)$ for which $|f(x) - f(x_0)| < \epsilon$ holds provided that $|x - x_0| < \delta$ for *any* $x_0 \in I$.
2. Find the radius of convergence of each of the following power series and find the exact interval of convergence (of course, give reasons for your answers)
 - a) $\sum \frac{x^n}{n^{n\sqrt{n}}}$
 - b) $\sum \frac{x^n}{n!}$
 - c) $\sum \frac{x^{2n+1}}{4^n \sqrt{n}}$
3. Let $f_n(x) = \frac{x}{n}$. Does $f_n(x)$ converge uniformly on
 - a) $[0, 1]$,
 - b) $[0, \infty)$ as $n \rightarrow \infty$?