Some definitions and theorems - Exam 3

1. Suppose that \( f \) is a function defined in an open interval containing the point \( c \). We say that \( c \) is a critical number of \( f \) if \( f \) is not differentiable at \( c \), or if \( f'(c) = 0 \).

2. Suppose that \( f \) is a function defined in an open interval \( I \). We say that a function \( F \) is an antiderivative (or a primitive) of \( f \) in \( I \) if \( F'(x) = f(x) \) for every \( x \) in \( I \).

3. Fermat’s Theorem Suppose that \( f \) is a function defined in an open interval, and let \( c \) be a point in the interval. If \( f \) has a local maximum or a local minimum at \( c \), then \( c \) is a critical number of \( f \).

4. Extreme Value Theorem Let \( a \) and \( b \) be real numbers with \( a < b \). Suppose that \( f \) is continuous on the closed interval \([a, b]\). Then there exist points \( p \) and \( q \) in \([a, b]\) such that \( f(p) \leq f(x) \leq f(q) \) for every \( a \leq x \leq b \).

5. Mean Value Theorem Let \( a \) and \( b \) be real numbers with \( a < b \). Suppose that \( f \) is continuous on the closed interval \([a, b]\), and that it is differentiable on the open interval \((a, b)\). Then there is some point \( c \) in \((a, b)\) such that

\[
\frac{f(b) - f(a)}{b - a} = f'(c).
\]

6. Fundamental Theorem of Calculus, Part I Let \( a \) and \( b \) be real numbers with \( a < b \). Suppose that \( f \) is continuous on the closed interval \([a, b]\). Define

\[
F(x) := \int_a^x f(t) \, dt, \quad a \leq x \leq b.
\]

Then \( F'(x) = f(x) \) for every \( a < x < b \), \( F'_+(a) = f(a) \), and \( F'_-(b) = f(b) \).

7. Fundamental Theorem of Calculus, Part II Let \( a \) and \( b \) be real numbers with \( a < b \). Suppose that \( f \) is continuous on the closed interval \([a, b]\). Let \( G \) be any function satisfying each of the following conditions: (i) \( G \) is continuous on the closed interval \([a, b]\), and (ii) \( G'(x) = f(x) \) for every \( a < x < b \). Then

\[
\int_a^b f(t) \, dt = G(b) - G(a).
\]