1. Suppose that \( f \) is a differentiable function. Evaluate each of the following limits:

\[
\lim_{h \to 0} \frac{f(a + 2h) - f(a)}{h} \quad \text{and} \quad \lim_{h \to 0} \frac{f(a + h) - f(a - h)}{h}.
\]

2. For what values of \( a \) and \( b \) is the line \( 2x + y = b \) tangent to the parabola \( y = ax^2 \) when \( x = 2 \)?

3. (taken from Basic Analysis: Japanese Grade 11) Two curves \( y = x^3 + ax \) and \( y = x^2 + bx + c \) pass through the point \((1,2)\) and have a common tangent line at this point. Find the values of the constants \( a \), \( b \), and \( c \).

4. Suppose that \( A \) is a fixed positive number.
   (a) Find an equation for each of the two tangent lines to the curve \( y = x^2 \) which pass through the point \((0,-A^2)\).
   (b) Find the value of \( A \) for which the aforesaid tangents are perpendicular to each other.

5. Let \( C \) denote the graph of the parabola \( y = 1 - x^2 \). Suppose that \( a \) is a (fixed) positive number, and let \( A \) denote the point \((0,1+a)\) on the \( y \)-axis. Suppose that the two tangent lines to \( C \) which pass through \( A \) touch \( C \) at the points \( P \) and \( Q \).
   (i) Find the co-ordinates of \( P \) and \( Q \) (in terms of \( a \)).
   (ii) Find the value of \( a \) such that the triangle formed by \( A \), \( P \), and \( Q \) is equilateral.

6. Suppose that \( f \) is differentiable at 0,

\[
\lim_{x \to 0} \frac{f(x)}{x} = 4, \quad \text{and} \quad \lim_{x \to 0} \frac{g(x)}{x} = 2.
\]

   (i) Find \( f(0) \).
   (ii) Find \( f'(0) \).
   (iii) Find \( \lim_{x \to 0} \frac{g(x)}{f(x)} \).

7. Suppose that \( f \) is a function that satisfies the equation

\[
f(x + y) = f(x) + f(y) + x^2y + xy^2
\]

for every pair of real numbers \( x \) and \( y \). Assume further that \( \lim_{x \to 0} \frac{f(x)}{x} = 1 \). Find (i) \( f(0) \), (ii) \( f'(0) \), and (iii) \( f'(x) \).
8. (from an old 151 (common) exam) Suppose that \( f \) is a differentiable function. It is known that the curve \( y = f(x) \) has exactly one horizontal tangent, corresponding to \( x = 2 \). Define \( g(x) = f(x^2 + x) \). Find all values of \( x \) for which the graph of \( g \) has horizontal tangents.

9. Suppose that \( f \) is a differentiable function such that \( f(1) = 1, f(2) = 2, f'(1) = 1, f'(2) = 2, \) and \( f'(3) = 3 \). If \( g(x) = f(x^3 + f(x^2 + f(x))) \), find \( g'(x) \) and hence \( g'(1) \).

10. Suppose \( a \) is a fixed positive number, and let \( P(x_0, y_0), y_0 \neq 0 \), be a fixed (but arbitrary) point on the astroid \( x^{2/3} + y^{2/3} = a^{2/3} \). Show that the length of the portion of the tangent line to the curve at \( P \) cut off by the co-ordinate axes is \( a \).

11. Suppose \( u(t) \) and \( v(t) \) are differentiable vector functions (of the real variable \( t \)). Show that

\[
\frac{d}{dt} [u(t) \cdot v(t)] = u(t) \cdot v'(t) + v(t) \cdot u'(t).
\]