1. If

\[ f(x) = \int_0^{\cos x} \frac{g(x)}{\sqrt{1 + t^3}} \, dt \quad \text{and} \quad g(x) = \int_0^{\sin t^2} \left[ 1 + \sin(t^2) \right] \, dt, \]

determine the value of \( f'(\pi/2) \).

2. Show that the function

\[ f(x) := \int_1^x \sqrt{1 + t^2} \, dt \]

is one-to-one. Compute \( (f^{-1})'(0) \).

The next pair of questions is taken from Basic Analysis: Japanese Grade 11, translated and published by the American Mathematical Society.

3. Find a (continuous) function \( f \) such that

\[ f(x) = x + \int_0^2 f(t) \, dt \]

for every real number \( x \).

4. Find a function \( f \) and a number \( a \) such that

\[ \int_1^x f(t) \, dt = x^3 + ax - 5 \]

for every real number \( x \).