1. (i) Determine the values of the number $p > 0$ for which the series

$$\sum_{n=1}^{\infty} \left[ 1 - \cos \left( \frac{1}{n} \right) \right]^p$$

is convergent/divergent.

(ii) Determine the values of the number $p > 0$ for which the series

$$\sum_{n=1}^{\infty} (-1)^{n+1} \left[ 1 - \cos \left( \frac{1}{n} \right) \right]^p$$

is conditionally convergent/absolutely convergent.

2. Suppose that $(a_n)$ is a sequence of positive terms, and that the series $\sum_{n=1}^{\infty} a_n$ is convergent. Show that the series $\sum_{n=1}^{\infty} a_n^k$ is convergent for every integer $k \geq 2$. 