Set 3


1. Compute the area enclosed between the curve $y = x^3$ and the tangent line to this curve at the point $(1, 1)$.

2. Let $L$ denote the straight line which passes through the origin, and is tangent to the curve $y = e^x$. Calculate the area enclosed by the said curve, $L$, and the $y$-axis.

3. Suppose $\alpha > 0$ is a fixed number, and let $P$ denote the point $(0, \alpha)$. Let $C$ denote the curve $y = -x^2$.
   (i) Find the equation(s) of the two tangents to $C$ which pass through $P$.
   (ii) Find the area enclosed by the curve $C$ and the two tangents (given above).

4. Suppose $m > 0$. Find all possible values of $m$ such that the line $y = mx$ and the curve $y = x/(x^2 + 1)$ enclose a region. Find the area of the enclosed region.

5. Suppose that a line intersects the parabola $y = x^2$ in two points $A$ and $B$ as shown in the figure below. Let $C$ be the point on the parabola where the tangent line is parallel to the line through $A$ and $B$. Show that the area of the parabolic segment cut off from the parabola by the line $AB$ is four-thirds the area of the triangle $ABC$.

![Diagram of parabola with points and lines]

6. The figure below shows a curve $C$ with the property that, for every point $P$ on the middle curve $y = 2x^2$, the areas $A$ and $B$ are equal. Find an equation for $C$.

![Diagram of curve C with points and lines]