1. Determine whether the following infinite integrals exist:
   
   (i) \( \int_0^\infty \frac{dx}{x^3 + 1} \)
   
   (ii) \( \int_0^\infty \frac{x}{x^2 + 1} \, dx \)
   
   (iii) \( \int_1^\infty \frac{3x^3 + x^2 + 5x + 2}{2x^5 + x^2 + 1} \, dx \)
   
   (iv) \( \int_2^\infty \frac{(x - 2)^2}{2x^{5/2} + x^2 + 3} \, dx \)

2. Suppose \( s > 0 \) is fixed, and let \( n \) be a positive integer.
   
   (i) Show that the infinite integral \( \int_0^\infty e^{-st^n} \, dt \) exists. (Use Limit Comparison or induction.)
   
   (ii) Show that
   \[ \int_0^\infty e^{-st^n} \, dt = \frac{n}{s} \int_0^\infty e^{-st^n-1} \, dt. \]
   
   (iii) Deduce that
   \[ \int_0^\infty e^{-st^n} \, dt = \frac{n!}{sn+1}. \]

3. Suppose \( n \) is a positive integer.
   
   (i) Show that
   \[ \int_0^1 (\ln x)^n \, dx = -n \int_0^1 (\ln x)^{n-1} \, dx. \]
   
   (ii) Deduce that
   \[ \int_0^1 (\ln x)^n \, dx = (-1)^n n!. \]

4. Let \( 0 < \alpha < 1 \) be fixed. Evaluate the improper integral
   \[ \int_{-2}^1 \frac{dx}{|x|^\alpha}. \]