MATH 407-10a, Assignment 10

Guidelines
Answer the questions in the space provided; you may write on both sides of the paper. Put the names of all group members in the top right corner. You may append additional sheets as needed. Please staple all sheets together before submission.

Due: Thursday, Feb 25th

1. (10 marks) Determine all possible values of the number

$$(1 - i)^{2i}.$$ 

What is its principal value?

2. (9 marks) Determine all possible complex numbers $z$ for which

$$\cos z = 0.$$ 

Definition. Let $Z_c$ denote the set of complex numbers determined in the preceding question, to wit,

$$Z_c := \{ z \in \mathbb{C} : \cos z = 0 \}.$$ 

The complex-valued tangent function is defined in the expected way:

$$\tan z := \frac{\sin z}{\cos z}, \quad z \in \mathbb{C} \setminus Z_c.$$

3. Let $S$ denote the infinite vertical strip given as follows:

$$S := \left\{ z \in \mathbb{C} : |\text{Re}(z)| \leq \frac{\pi}{4} \right\}.$$ 

(i) (10 marks) Prove: if $\omega \in S$, then $\tan \omega \in \overline{D}(0;1)$.

(ii) (6 marks) Suppose that $\omega \in S$. Show that $\tan \omega \in C(0;1)$ if and only if $|\text{Re}(\omega)| = \pi/4$. 