MATH 407-10a, Assignment 11

Guidelines
Answer the questions in the space provided; you may write on both sides of the paper. Put the names of all group members in the top right corner. You may append additional sheets as needed. Please staple all sheets together before submission.

Due: Tuesday, March 2nd

1. Consider the function
   \[ f(z) = (e^{-2xy} + e^{2xy}) \cos(x^2 - y^2) + i \left( e^{-2xy} - e^{2xy} \right) \sin(x^2 - y^2), \quad z = x + iy \in \mathbb{C}. \]
   (i) (10 marks) Verify that \( f \) is entire.
   (ii) (5 marks) Determine \( f'(z) \), \( z \in \mathbb{C} \) (in terms of the bivariate functions given above).

2. (10 marks) Suppose that \( f = u + iv \) is an entire function, and that \( f(0) = 1 - 2i \). Given that
   \[ u(x, y) = 2x^3 - 6xy^2 + x + 1, \]
   determine \( v \).