MATH 407-10a, Assignment 13

Guidelines
Answer the questions in the space provided; you may write on both sides of the paper. Put the names of all group members in the top right corner. You may append additional sheets as needed. Please staple all sheets together before submission.

Due: Tuesday, March 9th

1. (10 marks) Evaluate
\[ \int_{\gamma} (z^2 - 3|z| + \text{Im}(z)) \, dz, \]
where \( \gamma \) is the quarter circle (of radius 2) centred at the origin and extending from \( z = 2 \) to \( z = 2i \).

2. (10 marks) Let \( \gamma \) denote the elliptic arc parametrized by
\[ z(t) = 2 \cos t + 3i \sin t, \quad 0 \leq t \leq \frac{\pi}{2}. \]
Evaluate
\[ \int_{\gamma} z[\sin(\pi z^2) - \cos(\pi z^2)] \, dz. \]
Explain your steps clearly.

3. (10 marks) Suppose that \( \omega \) is a fixed (but arbitrary) complex number whose real part is nonnegative. Prove that
\[ |1 - e^{-\omega}| \leq |\omega|. \]
Explain your reasoning clearly and completely. (Integrate the function \( f(z) = e^{-z} \) along the straight line segment from \( z = 0 \) to \( z = \omega \).)