1. (20 marks) Suppose that $f : \mathbb{C} \to \mathbb{C}$ is a function, and that $f$ is continuous throughout the complex plane. Prove that the following are equivalent:

   (i) $f$ is entire.

   (ii) $\int_T f(z) \, dz = 0$ for every simple, positively-oriented triangle $T$ in the complex plane.

   (iii) $f$ admits a primitive throughout $\mathbb{C}$, that is, there is a function $F$ such that $F'(\omega) = f(\omega)$ for every complex number $\omega$.

(Prove that (i) implies (ii), (ii) implies (iii), and (iii) implies (i). To prove that (ii) implies (iii), let $\omega \in \mathbb{C}$, and let $\ell_\omega$ be the straight-line segment starting at $z = 0$ and ending at $z = \omega$. Define $F(\omega) := \int_{\ell_\omega} f(z) \, dz$.)