1. (7 marks) Prove the Minimum-Modulus Theorem: Let $f$ be analytic in an open connected set $D$. Assume that there is a point $z_0 \in D$ such that $|f(z)| \geq |f(z_0)| > 0$ for every $z \in D$. Then $f$ is constant in $D$.

2. Suppose that $g$ is analytic in an open connected set $D$.

   (i) (8 marks) Define $G(z) := \exp(g(z))$, $z \in D$. Prove that $g$ is constant in $D$ if $G$ is constant in $D$.

   (ii) (10 marks) Assume that there exists a point $z_0 \in D$ such that

   \[ \text{Re}(g(z)) \leq \text{Re}(g(z_0)) \quad \text{for every } z \in D. \]

   Prove that $g$ is constant in $D$. (Use (i).)