1. Consider the function
\[ f(z) = \frac{\sin(\pi z)}{(z - 1)^3(2z - 1)^2}. \]

(i) \(2 \text{ marks}\) Determine the singularities of \(f\).

(ii) \(12 \text{ marks}\) Compute the residue of \(f\) at each of its singularities.

2. \(6 \text{ marks}\) Let \(z \mapsto \sin z\) denote the complex-valued sine function (of the complex variable \(z\)).
   Verify that \(\sin z = 0\) if and only if \(z = k\pi\), \(k\) an integer.

In what follows \(\text{sinc}\) will denote the following function:
\[
\text{sinc}(z) := \begin{cases} 
\frac{\sin z}{z}, & \text{if } z \in \mathbb{C} \setminus \{0\}; \\
1, & \text{if } z = 0.
\end{cases}
\]

Recall that \(\text{sinc}\) is an entire function.

3. (i) \(4 \text{ marks}\) Show that the function \(\frac{1}{\text{sinc}}\), and hence the function \(\frac{1}{\text{sinc}^2}\), is analytic in \(D(0; \pi)\).

(ii) \(6 \text{ marks}\) Consider the function
\[ f(z) := \frac{e^z}{\sin^2 z}, \quad z \neq k\pi, \quad k \in \mathbb{Z}. \]
Compute the residue of \(f\) at \(z = 0\). (Write \(f\) in terms of the \(\text{sinc}\) function discussed above.)

(iii) \(5 \text{ marks}\) Evaluate
\[ \int_{\gamma} f(z) \, dz, \]
where \(\gamma\) denotes the circle \(C(0; 1)\) traversed once in the counterclockwise direction. (This question was set by Professor Roger Smith in his M407 final examination last term.)