1. Suppose that \( n \) is a fixed positive integer, and that \( z_1, \ldots, z_n \) are complex numbers. Use the principle of mathematical induction to prove each of the following:

   (i) (5 marks)
   \[
   \left( \sum_{k=1}^{n} z_k \right) = \sum_{k=1}^{n} \bar{z}_k
   \]

   (ii) (5 marks)
   \[
   (z_1 z_2 \cdots z_n) = \bar{z}_1 \bar{z}_2 \cdots \bar{z}_n
   \]

2. (10 marks) Let \( N \) be a fixed positive integer. Consider the polynomial

   \[
   P(z) := \sum_{k=0}^{N} a_k z^k, \quad z \in \mathbb{C},
   \]

   where every \( a_k \), \( 0 \leq k \leq N \), is a real number. Prove that a complex number \( \omega \) is a root of \( P \) if and only if \( \overline{\omega} \) is also a root of \( P \); that is, prove that \( P(\omega) = 0 \) if and only if \( P(\overline{\omega}) = 0 \).