- A nonempty subset G of $\mathbf{R}$ is said to be open if, for every $g \in G$, there is an $r > 0$ such that $(g - r_g, g + r_g) \subseteq G$.
- A subset of the real line is said to be closed if its complement is open.
- A subset of the real line is said to be compact if it is closed and bounded.
- Suppose that D is a nonempty set, and that $f : D \to \mathbf{R}$ is a function. Let $c \in D$. We say that f is continuous at c if the following holds: for every sequence $\{x_n\}_{n=1}^{\infty}$ in D such that $\lim_{n\to\infty} x_n = c$, we have $\lim_{n\to\infty} f(x_n) = f(c)$.
- Suppose that D is a nonempty set, and that $f : D \to \mathbf{R}$ is a function. We say that f is uniformly continuous on D if, given $\epsilon > 0$, there is a $\delta > 0$ such that $|f(s) - f(t)| < \epsilon$ whenever $s, t \in D$ and $|s - t| < \delta$.
- Suppose that f is a function defined in an open interval containing the number c. We say that f is differentiable at c if $\lim_{x\to c} \frac{f(x)-f(c)}{x-c}$ exists.

A set of necessary and sufficient conditions for a set to be closed:
Let F be a nonempty subset of $\mathbf{R}$. The following are equivalent:
(i) F is closed.
(ii) If $\{a_n\}_{n=1}^{\infty}$ is a sequence in F, and if $\lim_{n\to\infty} a_n = a$, then $a \in F$.

A set of necessary and sufficient conditions for a set to be compact:
Let K be a nonempty subset of $\mathbf{R}$. The following are equivalent:
(i) K is compact.
(ii) Every sequence in K admits a subsequence which converges to a point in K.

Nested Intervals Theorem: Suppose that $\{I_n\}_{n=1}^{\infty}$ is a sequence of closed intervals satisfying the following conditions:
(i) $I_{n+1} \subseteq I_n$ for every positive integer n.
(ii) $\lim_{n\to\infty} \text{length}(I_n) = 0$.
Then there is a unique point x^* which belongs to every I_n.

$\epsilon - \delta$ characterization of continuity at a point:
Let D be a nonempty set, and let $f : D \to \mathbf{R}$ be a function. Then f is continuous at a point $c \in D$ if and only if, given $\epsilon > 0$, there is a $\delta > 0$ such that $|f(x) - f(c)| < \epsilon$ whenever $|x - c| < \delta$.

Extreme Value Theorem: Let K be a nonempty compact subset of $\mathbf{R}$, and let $f : K \to \mathbf{R}$ be continuous on K. Then there exist points α_* and α^* in K such that $f(\alpha_*) \leq f(x) \leq f(\alpha^*)$ for every $x \in K$.

Intermediate Value Theorem: Let a and b be real numbers with $a < b$. Let $f : [a, b] \to \mathbf{R}$ be continuous on $[a, b]$. If T is any number between $f(a)$ and $f(b)$, then there is some c between a and b for which $f(c) = T$.