• A nonempty subset $G$ of $\mathbb{R}$ is said to be open if, for every $g \in G$, there is an $r > 0$ such that $(g - r, g + r) \subseteq G$.
• A subset of the real line is said to be closed if its complement is open.
• A subset of the real line is said to be compact if it is closed and bounded.
• Suppose that $D$ is a nonempty set, and that $f : D \to \mathbb{R}$ is a function. Let $c \in D$. We say that $f$ is continuous at $c$ if the following holds: for every sequence $\{x_n\}_{n=1}^{\infty}$ in $D$ such that $\lim_{n \to \infty} x_n = c$, we have $\lim_{n \to \infty} f(x_n) = f(c)$.
• Suppose that $D$ is a nonempty set, and that $f : D \to \mathbb{R}$ is a function. We say that $f$ is uniformly continuous on $D$ if, given $\epsilon > 0$, there is a $\delta > 0$ such that $|f(s) - f(t)| < \epsilon$ whenever $s, t \in D$ and $|s - t| < \delta$.
• Suppose that $f$ is a function defined in an open interval containing the number $c$. We say that $f$ is differentiable at $c$ if $\lim_{x \to c} \frac{f(x) - f(c)}{x - c}$ exists.

A set of necessary and sufficient conditions for a set to be closed:
Let $F$ be a nonempty subset of $\mathbb{R}$. The following are equivalent:
(i) $F$ is closed.
(ii) If $\{a_n\}_{n=1}^{\infty}$ is a sequence in $F$, and if $\lim_{n \to \infty} a_n = a$, then $a \in F$.

A set of necessary and sufficient conditions for a set to be compact:
Let $K$ be a nonempty subset of $\mathbb{R}$. The following are equivalent:
(i) $K$ is compact.
(ii) Every sequence in $K$ admits a subsequence which converges to a point in $K$.

Nested Intervals Theorem: Suppose that $\{I_n\}_{n=1}^{\infty}$ is a sequence of closed intervals satisfying the following conditions:
(i) $I_{n+1} \subseteq I_n$ for every positive integer $n$.
(ii) $\lim_{n \to \infty} \text{length}(I_n) = 0$.
Then there is a unique point $x^*$ which belongs to every $I_n$.

$\epsilon - \delta$ characterization of continuity at a point:
Let $D$ be a nonempty set, and let $f : D \to \mathbb{R}$ be a function. Then $f$ is continuous at a point $c \in D$ if and only if, given $\epsilon > 0$, there is a $\delta > 0$ such that $|f(x) - f(c)| < \epsilon$ whenever $|x - c| < \delta$.

Extreme Value Theorem: Let $K$ be a nonempty compact subset of $\mathbb{R}$, and let $f : K \to \mathbb{R}$ be continuous on $K$. Then there exist points $\alpha_*$ and $\alpha^*$ in $K$ such that $f(\alpha_*) \leq f(x) \leq f(\alpha^*)$ for every $x \in K$.

Intermediate Value Theorem: Let $a$ and $b$ be real numbers with $a < b$. Let $f : [a, b] \to \mathbb{R}$ be continuous on $[a, b]$. If $T$ is any number between $f(a)$ and $f(b)$, then there is some $c$ between $a$ and $b$ for which $f(c) = T$. 

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