Assignment 11

1. Consider the following sequences of functions:

(a)

$$f_n(x) := \frac{1}{1+nx}, \qquad n \in \mathbf{N}, \qquad x \in [0, \infty).$$

(b)

$$g_n(x) := \frac{1}{x+n}, \qquad n \in \mathbf{N}, \qquad x \in [0, \infty).$$

(c)

$$h_n(x) := \frac{x}{x+n}, \qquad n \in \mathbf{N}, \qquad x \in (0, \infty).$$

(i) In each of the three examples above, determine the pointwise limit of the given sequence, in the given domain.
(ii) Determine whether the convergence of each of the sequences in (i) is uniform on the domain. Justify your answers.

2. Suppose that $F : \mathbf{R} \to \mathbf{R}$ is uniformly continuous on $\mathbf{R}$. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence of nonzero real numbers such that $\lim_{n\to\infty} a_n = 0$. Define

$$F_n(x) := F(x + a_n), \qquad n \in \mathbf{N}, \qquad x \in \mathbf{R}.$$

Prove that the sequence $\{F_n\}$ converges to F uniformly on $\mathbf{R}$.