1. (i) Determine
\[ \lim_{n \to \infty} \frac{2n^2 + n + 1}{n^2 - n + 2}. \]
(ii) Give a formal proof to justify your assertion in (i).

2. This was discussed in lecture. Suppose that \( \{a_n\}_{n=1}^{\infty} \) is a sequence of real numbers, and let \( L \) be a fixed real number. Verify that the following statements are equivalent:
   (a) The sequence \( \{a_n\}_{n=1}^{\infty} \) does not converge to \( L \).
   (b) There exists a positive number \( \epsilon_0 \), and a sequence \( \{n_k : k \in \mathbb{N}\} \) of positive integers, such that \( n_k < n_{k+1} \) for every \( k \), and \( |a_{n_k} - L| \geq \epsilon_0 \) for every \( k \).
   (c) There exists a positive number \( \epsilon_0 \) such that \( |a_n - L| \geq \epsilon_0 \) for infinitely many values of \( n \).

3. Let \( a_n := (-1)^n, n \in \mathbb{N} \). Use the preceding result to show that \( \{a_n\}_{n=1}^{\infty} \) does not converge to any real number.

4. (i) Suppose that \( \{a_n\}_{n=1}^{\infty} \) is a sequence of real numbers, and that \( \lim_{n \to \infty} a_n = L \). Prove that \( \lim_{n \to \infty} |a_n| = |L| \).
   (ii) Give an example of a sequence \( \{a_n\}_{n=1}^{\infty} \) such that \( \lim_{n \to \infty} |a_n| \) exists, but \( \lim_{n \to \infty} a_n \) does not exist.

5. Suppose that \( \{a_n\}_{n=1}^{\infty} \) is a sequence of real numbers. Prove that the following statements are equivalent:
   (a) \( \lim_{n \to \infty} a_n = 0 \).
   (b) \( \lim_{n \to \infty} |a_n| = 0 \).
   (c) \( \lim_{n \to \infty} a_n^2 = 0 \).

6. (i) Suppose that \( \{c_n\}_{n=1}^{\infty} \) is a sequence of real numbers satisfying the following condition: there exists a positive integer \( n_0 \) such that \( c_n \geq 0 \) for every \( n \geq n_0 \). Assume further that \( \lim_{n \to \infty} c_n = L \). Prove that \( L \geq 0 \). (Suppose that \( L < 0 \) and derive a contradiction.)
   (ii) Give an example of a sequence of positive numbers which converges to zero.
   (iii) Suppose that \( \{a_n\}_{n=1}^{\infty} \) and \( \{b_n\}_{n=1}^{\infty} \) are sequences satisfying the following condition: there exists a positive integer \( n_0 \) such that \( a_n \geq b_n \) for every \( n \geq n_0 \). Assume further that \( \lim_{n \to \infty} a_n = A \) and \( \lim_{n \to \infty} b_n = B \). Use (i) to show that \( A \geq B \).

7. Questions from the text:
   7.1(c), (d); 7.2 for these sequences
   7.3(e), (g), (h), (i), (m), (p)
   7.4, 7.5(a), 8.8(a)
   8.4, 8.7(b)