MATH 409-08a, Quiz 1

Guidelines

1. You may use your class notes and text, but nothing else.
2. If you choose to work alone, you may not consult anyone except your course intstructor; if you are working with a partner (no more than one partner allowed), you may consult no one other than your partner or your course instructor.
3. Answer the questions in the space provided; you may write on both sides of the paper. Put your name (two names when applicable) in the top right corner. You may append additional sheets as needed, but if you do, *staple everything together before submission.* Write neatly and legibly; shoddy presentation may lead to appropriate penalization.

Due: Friday, January 25th (by 4:00 pm)

1. (i) (8 marks) Show that

$$||a|-|b|| \leq |a-b|$$

for every pair of real numbers a and b.
(ii) (8 marks) Suppose that x, y, and z are (arbitrary) real numbers. Prove that

$$|x-y-z| \geq |x|-|y|-|z|.$$

2. (9 marks) Suppose that S and T are nonempty, bounded subsets of the real line. Prove that

$$\inf(S \cup T) = \min\{\inf(S), \inf(T)\}.$$

(1) (i) $|a| = |a-b+b| \underset{\Delta \text{ ineq}}{\leq} |a-b| + |b|$

$\Rightarrow \boxed{|a|-|b| \leq |a-b|}$ ①

Similarly, $|b| = |b-a+a| \leq |b-a| + |a|$

$\Rightarrow \boxed{|b|-|a| \leq |b-a| = |a-b|}$ ②

From ① + ② we see that $\pm(|a|-|b|) \leq \underbrace{|a-b|}_{\geq 0}$

$\Rightarrow ||a|-|b|| \leq |a-b|.$

(ii) Use (i) twice:

$|x-y-z| = |(x-y)-z| \overset{(i)}{\geq} |x-y| - |z|$

$\underset{(i)}{\geq} |x|-|y|-|z|.$

$\boxed{|a-b| \geq ||a|-|b|| \geq |a|-|b|}$

(2) Let $x \in S \cup T$. $\Rightarrow x \in S$ or $x \in T$.

If $x \in S$, then $x \geq \inf(S) \geq \min\{\inf(S), \inf(T)\}$.

If $x \in T$, then $x \geq \inf(T) \geq \min\{\inf(S), \inf(T)\}$.

$\therefore x \geq \min\{\inf(S), \inf(T)\} \quad \forall x \in S \cup T$

$\Rightarrow \min\{\inf(S), \inf(T)\}$ is a lower bound for $S \cup T$. $\therefore \boxed{\inf(S \cup T) \geq \min\{\inf(S), \inf(T)\}.}$ (3)

Now if $x \in S$, then $x \in S \cup T$, so $x \geq \inf(S \cup T)$. $\therefore \inf(S \cup T)$ is a lower bound for S. $\therefore \inf(S) \geq \inf(S \cup T)$.

The same argument shows that $\inf(T) \geq \inf(S \cup T)$.

$\therefore \inf(S \cup T) \leq \inf(S)$ and $\inf(T)$

$\Rightarrow \boxed{\inf(S \cup T) \leq \min\{\inf(S), \inf(T)\}}$ (4)

(3) + (4) $\Rightarrow$ required result.