SOLUTIONS

MATH 409-08a, Quiz 2

Guidelines

1. You may not use any instructional aids other than your text and lecture notes.
2. If you choose to work alone, you may not consult anyone except your course intstructor; if you are working with a partner (no more than one partner allowed), you may consult no one other than your partner or your course instructor.
3. Answer the questions in the space provided; you may write on both sides of the paper. Put your name (two names when applicable) in the top right corner. You may append additional sheets as needed, but if you do, *staple everything together before submission*. Write neatly and legibly; shoddy presentation may lead to appropriate penalization.

Due: Friday, February 1st (by 4:00 pm)

1. (8 marks) Suppose that S and T are subsets of the real line, that $S \subseteq T$, and that S is nonempty. Shew that $\mathrm{glb}(T) \leq \mathrm{glb}(S)$.

2. (8 marks) Let S be a nonempty subset of the real line, and assume that S is bounded above. Let $\alpha > 0$ be a fixed number, and define

$$\alpha S := \{\alpha s : s \in S\}.$$

Show that $\sup(\alpha S) = \alpha \sup(S)$.

3. (9 marks) Suppose that $\{c_n\}_{n=1}^{\infty}$ is a sequence of nonnegative real numbers, and that $\lim_{n\to\infty} c_n = L$. Prove that $L \geq 0$.

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1) Case 1. S not bdd below.

In this case, T cannot be bdd below either.

so $\inf(T) = -\infty = \inf(S)$

Case 2. S is bounded below, but T is not.

Here $\inf(T) = -\infty < \inf(S)$ [which is some finite real #]

Case 3. S and T are bdd below.

Now $\inf(T)$ is a lower bound for T, so it is also a lower bound for S, because $S \subseteq T$.

$\therefore$ $\inf(T) \leq \inf(S)$, because the latter is the greatest lower bound for S.

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2) Let $\sup(S) =: \Delta$. As $s \leq \Delta \ \forall s \in S$ and $\alpha > 0$, $\alpha s \leq \alpha\Delta \ \forall s \in S$. $\therefore$ $\alpha\Delta$ is an upper bound for αS. We now show that $\alpha\Delta$ is the lub for αS. To that end,

let $y < \alpha\Delta$. Then $\frac{y}{\alpha} < \Delta$. As $\Delta = \text{lub}(S)$, $\exists\, s_0 \in S$ s.t. $\frac{y}{\alpha} < s_0 \Rightarrow y < \alpha s_0 \in \alpha S$. Thus we have found an element in αS which is larger than y. So y cannot be an upper bound for αS. This being true for all $y < \alpha\Delta$, we conclude that $\alpha\Delta = \sup(\alpha S)$.

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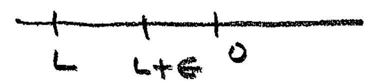
(3) Assume that $L < 0$. Choose $\epsilon > 0$ such that $L + \epsilon < 0$. As



$\{c_n\} \longrightarrow L$, $\exists\, N \in \mathbb{N}$ s.t. $L - \epsilon < c_n < L + \epsilon$ $\forall\, n \geq N$. But this means, in particular, that $c_n < 0$ $\forall\, n \geq N$, a clear contradiction. The result follows.
