1. As $f$ is continuous at $c$, we know that, given $\epsilon > 0$, there is a $\delta > 0$ such that $-\epsilon < f(x) - f(c) < \epsilon$ whenever $|x - c| < \delta$. Choose $\epsilon = 5/4$, and denote the corresponding $\delta$ by $\delta_0$. We may choose $\delta_0$ sufficiently small so that $a < c - \delta_0 < c < c + \delta_0 < b$. Let $I := (c - \delta_0, c + \delta_0)$. So if $x \in I$, then $-5/4 < f(x) - 3 < 5/4$, which implies, in particular, that $f(x) > 7/4$.

2. Assume that $f$ is continuous on $\mathbb{R}$, and let $G$ be a nonempty open set in $\mathbb{R}$. We must show that $I_f(G) := \{x \in \mathbb{R} : f(x) \in G\}$ is open. If this set is empty, there is nothing to prove, so assume that it is not. We must therefore show that, if $p \in I_f(G)$, then there is some $r > 0$ such that $(p - r, p + r) \subseteq I_f(G)$. Now $p \in I_f(G)$ implies that $f(p) \in G$. As $G$ is open, there is some $\epsilon > 0$ such that $(f(p) - \epsilon, f(p) + \epsilon) \subseteq G$. As $f$ is continuous at $p$, there is some $r > 0$ such that $|f(x) - f(p)| < \epsilon$ whenever $x \in (p - r, p + r)$. In other words, $f(p) - \epsilon < f(x) < f(p) + \epsilon$ whenever $x \in (p - r, p + r)$. This implies that $f(x) \in (f(p) - \epsilon, f(p) + \epsilon) \subseteq G$ whenever $x \in (c - r, c + r)$. Thus $f(x) \in G$ for every $x \in (c - r, c + r)$; that is, $(c - r, c + r) \subseteq I_f(G)$.

Conversely, assume that $I_f(G)$ is open whenever $G$ is. We must show that $f$ is continuous at every point in $\mathbb{R}$. Let $c \in \mathbb{R}$, and let $\epsilon > 0$ be given. Define $G := (f(c) - \epsilon, f(c) + \epsilon)$. Clearly $G$ is open, so $I_f(G)$ is also open by assumption. As $f(c) \in G$, $c \in I_f(G)$, so there is some $\delta > 0$ such that $(c - \delta, c + \delta) \subseteq G$. This implies that $f(x) \in G$ whenever $x \in (c - \delta, c + \delta)$, or that $f(c) - \epsilon < f(x) < f(c) + \epsilon$ whenever $c - \delta < x < c + \delta$. It follows that $f$ is continuous at $c$. 