

MATH 409-08a, Quiz 10

Guidelines

1. You may not use any instructional aids other than your text and lecture notes.
2. If you choose to work alone, you may not consult anyone except your course instructor; if you are working with a partner (no more than one partner allowed), you may consult no one other than your partner or your course instructor.
3. Answer the questions in the space provided; you may write on both sides of the paper. Put your name (two names when applicable) in the top right corner. You may append additional sheets as needed, but if you do, *staple everything together before submission*. Write neatly and legibly; shoddy presentation may lead to appropriate penalization.

Due: Thursday, April 17th

1. (9 marks) Let $f(x) := x^{111} + x^{11} + x + 1$. Prove that there is a unique number p such that $f(p) = 0$. (First show that there is at least one such p ; then prove that there can be at most one.)
2. (8 marks) Prove that $|\cos u - \cos v| \leq |u - v|$ for every pair of real numbers u and v .
3. (8 marks) Suppose that $f : \mathbf{R} \rightarrow \mathbf{R}$ is a differentiable function, and that $f'(x) < 1$ for every real number x . Prove that f can have at most one fixed point (that is, at most one number t such that $f(t) = t$). (Assume that f has more than one fixed point and obtain a contradiction.)