MATH 409-08a, Quiz 11

Guidelines

1. You may not use any instructional aids other than your text and lecture notes.

2. If you choose to work alone, you may not consult anyone except your course intstructor; if you are working with a partner (no more than one partner allowed), you may consult no one other than your partner or your course instructor.

3. Answer the questions in the space provided; you may write on both sides of the paper. Put your name (two names when applicable) in the top right corner. You may append additional sheets as needed, but if you do, *staple everything together before submission*. Write neatly and legibly; shoddy presentation may lead to appropriate penalization.

Due: Thursday, April 24th

1. (`9 marks`) Mimic the proof of the special case done in lecture to finish the proof of the general case of Taylor's Remainder Theorem: Suppose that n is a natural number, that f and its derivatives $f', f'', \ldots, f^{(n-1)}$ are defined and continuous on the interval $[a, b]$, and that $f^{(n)}$ exists in (a, b). If $\alpha, \beta \in [a, b]$, then there is a number γ between α and β such that

$$f(\beta) = \sum_{k=0}^{n-1} \frac{f^{(k)}(\alpha)}{k!}(\beta - \alpha)^k + \frac{f^{(n)}(\gamma)}{n!}(\beta - \alpha)^n.$$

2. (i) (`8 marks`) Suppose that f is continuous on $[a, b]$. Let c be a point in $[a, b]$ such that $f(c) = \alpha > 0$. Prove that there is a closed interval J such that $c \in J \subseteq [a, b]$, and $f(x) > \alpha/2$ for every $x \in J$.

(ii) (`8 marks`) Suppose that f is continuous on the interval $[a, b]$, that $f(x) \geq 0$ for every $a \leq x \leq b$, and that $\int_a^b f = 0$. Prove that $f(x) = 0$ for every $x \in [a, b]$. (Assume that f is strictly positive at some point, and use (i) to get a contradiction.)