MATH 409-08a, Quiz 3

Guidelines

1. You may not use any instructional aids other than your text and lecture notes.
2. If you choose to work alone, you may not consult anyone except your course intstructor; if you are working with a partner (no more than one partner allowed), you may consult no one other than your partner or your course instructor.
3. Answer the questions in the space provided; you may write on both sides of the paper. Put your name (two names when applicable) in the top right corner. You may append additional sheets as needed, but if you do, *staple everything together before submission*. Write neatly and legibly; shoddy presentation may lead to appropriate penalization.

Due: Friday, February 8th (by 4:00 pm)

1. Suppose that $\lim_{n\to\infty} a_n = +\infty$, and let $\{b_n\}_{n=1}^{\infty}$ be a convergent sequence of positive numbers.
  (i) (`8 marks`) Prove: if $\lim_{n\to\infty} b_n = b > 0$, then $\lim_{n\to\infty} a_n b_n = +\infty$.
  (ii) (`8 marks`) Show by means of an example that the assertion in (i) need not hold if $b = 0$.

2. (`9 marks`) Suppose that S is a nonempty subset of the real line, and assume that S is bounded above by α. Prove that the following statements are equivalent:
  (a) $\alpha = \sup(S)$.
  (b) There exists a sequence in S which converges to α.
  (To show that (a) implies (b), consider $\alpha - \frac{1}{n}$ for every positive integer n.)