

MATH 409-08a, Quiz 5

Guidelines

1. You may not use any instructional aids other than your text and lecture notes.
2. If you choose to work alone, you may not consult anyone except your course instructor; if you are working with a partner (no more than one partner allowed), you may consult no one other than your partner or your course instructor.
3. Answer the questions in the space provided; you may write on both sides of the paper. Put your name (two names when applicable) in the top right corner. You may append additional sheets as needed, but if you do, *staple everything together before submission*. Write neatly and legibly; shoddy presentation may lead to appropriate penalization.

Due: Friday, February 22nd (by 4:00 pm)

1. (i) (7 marks) Suppose that $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ are bounded sequences. Prove that

$$\limsup_{n \rightarrow \infty} (a_n + b_n) \leq \limsup_{n \rightarrow \infty} a_n + \limsup_{n \rightarrow \infty} b_n.$$

- (ii) (4 marks) Show by means of an example that strict inequality can occur in (i) above; that is, find bounded sequences $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ for which

$$\limsup_{n \rightarrow \infty} (a_n + b_n) < \limsup_{n \rightarrow \infty} a_n + \limsup_{n \rightarrow \infty} b_n.$$

2. (14 marks) Suppose that $\{a_n\}_{n=1}^{\infty}$ is a bounded sequence. Prove that the following statements are equivalent:
 - (a) $m = \liminf_{n \rightarrow \infty} a_n$.
 - (b) For every $\epsilon > 0$, $a_n > m - \epsilon$ for all but at most finitely many values of n , and $a_n < m + \epsilon$ for infinitely many values of n .