Summary

Nice figures of regions in the plane may be rendered with MATLAB via `fill` and `plot` commands. The `fill` command paints the interior of the region with a solid color, whereas `plot` is used to draw the boundary of the region. Illustrate intersections of boundary pieces by plotting individual points (using filled in circles for the marker type).

- In Chapter 3 of Gilat, review mathematical operations with arrays. (Use the `vectorize` command to make element-by-element operations easy!)
- Concatenate vectors to join boundary pieces.
- The `fliplr` command is useful for traversing the closed boundary in a counterclockwise fashion.

MATLAB Figures

MATLAB diary files (input and output via `echo on`) appear on the reverse. Below just look at the pretty pictures!

1. First paint the area with `fill`.
2. Then draw the boundary with `plot` command(s).
3. Highlight intersections with `plot` using filled in circles for the marker type.
4. Add axes, labels, annotation as desired.

1. Cartesian boundary functions $x = g(y)$

We draw and find the area of the region enclosed by the parabolas $x = 2y - y^2$ and $x = y^2 - 4y$; area: 9 cm^2.

2. Cartesian boundary functions $y = f(x)$

We draw and find the area of the region enclosed by the parabola $y = (x - 2)^2$ and line $y = x$; area: 4.5 cm^2.

3. Parametric boundary curve $r(t) = [x(t), y(t)]$

An ellipse $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ with center $(h,k)$ and semi-axis lengths $a$ and $b$ may be parameterized in terms of $t$ as $x = h + a \cos t$, $y = k + b \sin t$, $0 \leq t \leq 2\pi$. [Note that if $a = b$ we have a circle of radius $a$ with center $(h,k)$.]

4. Regions bounded by Cartesian curve, coordinate axis, and tangent/normal line

Find the area of the region bounded by the parabola $y = x^2$, the normal line to the parabola at $(2,4)$, and the $x$-axis. (The normal line to a curve $C$ at a point $P$ is the line through $P$ that is perpendicular to the tangent line at $P$. The slopes of these two lines are negative reciprocals.)
% Supplement 7: Problem 1
% Area of region bounded by parabolas
syms y % symbolics
x1 = 2*y - y^2;
x2 = y^2 - 4*y;
y_at_intersections = solve(x1 == x2, y)
y_at_intersections =
0
3
A = int(x1-x2, y, 0, 3) % cm^2
A =
9
% PLOT OF REGION USING AREA FILL!
y = linspace(0,3);
s = vectorize(x1)
s =
2.*y - y.^2
x1 = eval(s); % convert to numbers
x2 = eval(vectorize(x2)); % Or nest 'em!
X = [x1 fliplr(x2)]; % Traverse boundary counterclockwise.
Y = [y fliplr(y)]; % corresponding y-values
fill(X,Y,'y'); grid on; hold on
plot(X,Y, 'LineWidth', 2)
plot([-5 2], [0 0], 'k') % Embellishments
plot([0 0], [-3 5], 'k')
plot([2 2], [-3 5], 'k--')
plot(2,1,'ko', 'MarkerFaceColor', 'k')
xlabel('x'); ylabel('y')
title('Supplement 7: Problem 2')
axis equal; axis([-1 5 -1 5])
set(gca, 'Xtick', -1:5)
set(gca, 'Ytick', -1:5)
% echo off; diary off

% Supplement 7, Problem 3
% t = linspace(0,2*pi);
x = 2 + 4*cos(t);
y = 1 + 2.5*sin(t);
fill(x,y,'r'); grid on; hold on
plot(x,y, 'LineWidth', 3)
plot([-3 7], [0 0], 'k')
plot([0 0], [-3 5], 'k')
plot([-3 7], [1 1], 'k--')
plot([2 2], [-3 5], 'k--')
title('Supplement 7, Problem 3')
axis equal; axis([-3 3 -3 5])
set(gca, 'Xtick', [-3:3])
set(gca, 'Ytick', [-3:5])
% echo off; diary off