1. (a) [5 points] A particle starts at the origin with initial velocity \( \mathbf{v} = \mathbf{i} - \mathbf{j} + 3\mathbf{k} \). Its acceleration is 
\[ a(t) = (6t) \mathbf{i} + (12t^2) \mathbf{j} + (-6t) \mathbf{k} \]. Find its position function.
   - Antidifferentiate acceleration to obtain velocity, then velocity to obtain position, resolving vector constants of integration along the way.
   - \[ \mathbf{v}(t) = \int a(t) \, dt = \left[ 6t, 12t^2, -6t \right] + \mathbf{C} \]
   - \[ \mathbf{r}(t) = \int \mathbf{v}(t) \, dt = \left[ 3t^2 + 1, 4t^3 - 1, -3t^2 + 3 \right] + \mathbf{K} \]
   - \[ \mathbf{r}(0) = \mathbf{K} = [0, 0, 0] \]

(b) [5 points] Find a vector that has the same direction as the vector \( \mathbf{v} = [-2, 4, 2] \) but whose magnitude is 6.
   - The vector \( \mathbf{w} = 6 \hat{v} \) fits the bill.
   - \[ \mathbf{w} = 6 \frac{\mathbf{v}}{||\mathbf{v}||} = \frac{6[-2, 4, 2]}{\sqrt{4 + 16 + 4}} = \frac{-12, 24, 12}{\sqrt{24}} = [-\sqrt{6}, 2\sqrt{6}, \sqrt{6}] \]

2. [15 points] Find the work done by a force \( \mathbf{F} = 8\mathbf{i} - 6\mathbf{j} + 9\mathbf{k} \) that moves an object from the point \( P(0, 10, 8) \) to the point \( Q(6, 12, 20) \) along a straight line. The distance is measured in meters and the force in newtons.
   - The displacement vector is \( \mathbf{D} = \overrightarrow{PQ} = \mathbf{Q} - \mathbf{P} = [6, 2, 12] \).
   - The work done is \( W = \mathbf{F} \cdot \mathbf{D} = 48 - 12 + 108 = 144 \text{ N-m or 144 joules} \).

3. [15 points] Find the volume of the parallelepiped (slanted box) determined by the vectors \( \mathbf{a} = [6, 3, -1] \), \( \mathbf{b} = [0, 1, 2] \), and \( \mathbf{c} = [4, -2, 5] \). Lengths are in centimeters.
   - The volume is given by the absolute value of the scalar triple product of \( \mathbf{a}, \mathbf{b}, \) and \( \mathbf{c} \).
   - \[ |\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})| = |[6, 3, -1] \cdot [9, 8, -4]| = 54 + 24 + 4 = 82 \text{ cm}^3 \]

4. [15 points] Find an equation of the plane that passes through the point \( P(6, 0, -2) \) and contains the line \( x = 4 - 2t, y = 3 + 5t, z = 7 + 4t \).
   - Let’s find two points on the line \( \mathbf{L}(t) = [4 - 2t, 3 + 5t, 7 + 4t] \). Here are their position vectors.
     \[ \mathbf{Q} = \mathbf{L}(0) = [4, 3, 7] \]
     \[ \mathbf{R} = \mathbf{L}(1) = [2, 8, 11] \]
   - A normal vector to the plane is
     \[ \mathbf{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = (\mathbf{Q} - \mathbf{P}) \times (\mathbf{R} - \mathbf{P}) = [-2, 3, 9] \times [-4, 8, 13] = [-33, -10, -4] \]
   - Now construct an equation of the plane.
     \[ \mathbf{n} \cdot (x, y, z) = \mathbf{n} \cdot \mathbf{P} \]
     \[ -33x - 10y - 4z = -190 \text{, or } 33x + 10y + 4z = 190 \]
5. [15 points] Find parametric equations for the tangent line to the curve \( \mathbf{r}(t) = [1 + 2\sqrt{t}, t^3 - t, t^3 + t] \) at the point \( A (3, 0, 2) \).

- The value of the parameter corresponding to \( A \) is \( t = 1 \).
- A direction vector of the tangent line is the tangent vector \( \mathbf{v} = \mathbf{r}'(1) \). Now
  \[
  \mathbf{r}'(t) = \begin{bmatrix} t^{-1/2} & 3t^2 - 1 & 3t^2 + 1 \end{bmatrix}
  \]
  whence \( \mathbf{v} = \mathbf{r}'(1) = [1, 2, 4] \).
- Now form an equation of the line.
  \[
  \mathbf{L}(u) = \mathbf{A} + u\mathbf{v}
  \]
  \[
  [x(u), y(u), z(u)] = [3, 0, 2] + u[1, 2, 4]
  \]
  \[
  [x, y, z] = [u + 3, 2u, 4u + 2]
  \]

6. [15 points] Find the arc length of the curve \( \mathbf{r}(t) = [t, \frac{1}{2}t^2, \frac{1}{3}t^3] \), \( 0 \leq t \leq 6 \).

- First compute the vector derivative: \( \mathbf{r}'(t) = [1, t, \frac{1}{3}t^2] \).
- Next, determine its magnitude: \( \| [1, t, \frac{1}{3}t^2] \| = \sqrt{1 + t^2 + \frac{1}{9}t^4} = 1 + \frac{1}{3}t^2 \).
- Finally, compute the arc length:
  \[
  L = \int_{a}^{b} \| \mathbf{r}'(t) \| \, dt = \int_{0}^{6} 1 + \frac{1}{3}t^2 \, dt = \left( t + \frac{1}{9}t^3 \right)_{0}^{6} = (6 + 36) - (0) = 42.
  \]

7. [15 points] The position function of a particle is given by \( \mathbf{r}(t) = [t^2, 5t, t^2 - 16t] \), \( 0 \leq t \leq 10 \), where distance is in centimeters and time is in seconds. Find the minimum and maximum speeds of the particle over the stated time and when they occur.

- First compute the velocity, a vector quantity: \( \mathbf{v}(t) = \mathbf{r}'(t) = [2t, 5, 2t - 16] \).
- Next determine the speed, a scalar quantity: \( f(t) = \| \mathbf{v}(t) \| = \sqrt{4t^2 + 25 + 4t^2 - 64t + 256} \).
- While we could use calculus (which is fine), it turns out that algebra will actually suffice. Toward this end, let’s obtain an alternative expression for the speed by completing the square!
  \[
  f(t) = \sqrt{8t^2 - 64t + 281} = \sqrt{8(t^2 - 8t + 16) + 281 - 128} = \sqrt{8(t - 4)^2 + 153}.
  \]
- Now it’s easy to see that the minimum speed occurs at \( t = 4 \) seconds.
  \[
  f(4) = \sqrt{153} = 3\sqrt{17} \approx 12.37 \text{ cm/s}
  \]
- Moreover, the maximum speed occurs at \( t = 10 \) seconds.
  \[
  f(10) = \sqrt{441} = 21 \text{ cm/s}
  \]
- Alternatively, solve \( f''(t) = \frac{8(t - 4)}{\sqrt{8t^2 - 64t + 281}} = 0 \) to obtain \( t = 4 \in [0, 10] \). Then use the Extreme Value Theorem: evaluate \( f \) at this interior point and at the endpoints.

<table>
<thead>
<tr>
<th>( t ) [sec]</th>
<th>( f(t) ) [cm/s]</th>
<th>Comment</th>
</tr>
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<tr>
<td>0</td>
<td>( \sqrt{281} \approx 16.76 )</td>
<td>intermediate value</td>
</tr>
<tr>
<td>4</td>
<td>( \sqrt{153} \approx 12.37 )</td>
<td>absolute minimum</td>
</tr>
<tr>
<td>10</td>
<td>( \sqrt{441} = 21 )</td>
<td>absolute maximum</td>
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