12.7: An Unconstrained Max/Min Problem via TI-89/TAMUCALC Commands

In the TI-89 command sequences contained herein, spaces are inserted for legibility. You may leave them out.

c161x05

Find and classify all critical points of \( f(x, y) = x^4 - 2x^2 - y^3 + 3y \). Corroborate with graphics on your own!

1. Solve \( \nabla f = 0 \), obtain critical points.

- \( \text{mf}([x,y], x^4 - 2x^2 - y^3 + 3y, "f") \) yields \( x^4 - 2x^2 - y^3 + 3y \). The colon (:) separates commands on the entry line.
- \( \text{grad}(f(x,y), [x,y]) \) yields \( 4x^3 - 4x, 3 - 3y^2 \), the gradient of \( f \).
- \( \text{equate(ans(1), [0,0])} \) yields \( (x^2 - 1)x = 0 \) and \( y^2 = 1 \), the result of setting the gradient of \( f \) equal to the zero vector and subsequent automatic simplification. (NOTE: You may enter 0 instead of a zero vector for convenience.)
- Finally, \( \text{solve(ans(1), f(x,y))} \) reveals six (6) critical points, \( (x, y) = (1, 1), (1, -1), (0, 1), (0, -1), (-1, 1), (-1, -1) \).

2. Compute the Hessian matrix and make it into a function. Then use the SDT (Second Derivative Test) on each critical point.

- \( \text{hess}(f(x,y), [x,y]) \) returns \[
\begin{bmatrix}
12x^2 - 4 & 0 \\
0 & -6y
\end{bmatrix}
\] , then \( \text{mf}([x,y], \text{ans(1)}, "h") \) makes a Hessian matrix function named \( h \).
- To analyze a critical point, say \( (-1, -1) \), compute the Hessian matrix thereat, \( h(-1,-1) \) giving \[
\begin{bmatrix}
8 & 0 \\
0 & 6
\end{bmatrix}
\] , then the LPMDs (Leading Principal Minor Determinants) via \( \text{LPMD}(h(1,1)) \) or \( \text{LPMD}(\text{ans(1)}) \), yielding \{8, 48\}. This signifies a local minimum with \( f(-1,-1) \) being \(-3\).
- Here is the table we constructed in class analyzing all six critical points. You can verify the cell entries with your TI-89 in a manner similar to the analysis used for the \( (-1, -1) \) critical point.

<table>
<thead>
<tr>
<th>((x, y))</th>
<th>(f(x, y))</th>
<th>(H(x, y))</th>
<th>LPMDs</th>
<th>Classification</th>
</tr>
</thead>
</table>
| \((-1, -1)\) | \(-3\) | \[
\begin{bmatrix}
8 & 0 \\
0 & 6
\end{bmatrix}
\] | \{8, 48\} | local minimum |
| \((-1, 1)\) | 1 | \[
\begin{bmatrix}
8 & 0 \\
0 & -6
\end{bmatrix}
\] | \{8, -48\} | saddle point |
| \((0, -1)\) | \(-2\) | \[
\begin{bmatrix}
-4 & 0 \\
0 & 6
\end{bmatrix}
\] | \{-4, -24\} | saddle point |
| \((0, 1)\) | 2 | \[
\begin{bmatrix}
-4 & 0 \\
0 & -6
\end{bmatrix}
\] | \{-4, 24\} | local maximum |
| \((1, -1)\) | \(-3\) | \[
\begin{bmatrix}
8 & 0 \\
0 & 6
\end{bmatrix}
\] | \{8, 48\} | local minimum |
| \((1, 1)\) | 1 | \[
\begin{bmatrix}
8 & 0 \\
0 & -6
\end{bmatrix}
\] | \{8, -48\} | saddle point |