Section 1.1

1. [9/20e] Draw solution curves for the differential equation \( \frac{dy}{dt} = y = y (y - 3) \) for \(-5 \leq t \leq 5\), \(-5 \leq y \leq 5\). Classify any equilibrium solutions. The equation is autonomous (the derivative only depends on the dependent variable \( y \), not on the independent variable \( t \)). The plot is easily drawn by hand. Verify with the Java routine dfield or your TI-Nspire CX CAS calculator (see video).

2. [9/24] A drug is administered intravenously. Fluid containing 5 mg/cm\(^3\) of the drug enters the patient’s bloodstream at a rate of 100 cm\(^3\)/h. The drug is absorbed by the body tissues or otherwise leaves the bloodstream at a rate proportional to the amount present, with a rate constant of \( r = 0.4 \text{ h}^{-1} \).

   (a) Write a differential equation for the amount of drug present in the bloodstream at any time.

   (b) How much of the drug is present in the bloodstream after a long time?

3. [10/29] Employ MATLAB to draw a direction field and solution curves for the differential equation \( y' = t + 3y \), which is nonautonomous. Use my routine dfic and the command dsolve. Describe how solutions behave as \( t \to \infty \).

4. [10/32] Draw a direction field and a few solution curves for \( y' = -\frac{2t + y}{2y} \). You may use dfield or your calculator (see video).

Section 1.2

1. [16/2a] Solve \( \frac{dy}{dt} = y = y - 5 \), \( y (0) = y_0 \) (an initial value problem). Plot solutions for several values of \( y_0 \). Describe how solutions resemble, and differ from, each other.

2. [17/8] Consider a population of field mice that grows at a rate proportional to the current population, so that \( \frac{dp}{dt} = p' = rp \). (b) Find the rate constant \( r \) if the population doubles in \( N \) days. (a) Find \( r \) if \( N = 30 \).

Section 1.3

1. [24/2] Determine the order of the differential equation \( (1 + y^2) \frac{d^2y}{dt^2} + t \frac{dy}{dt} + y = e^t; \) also state whether it is linear or nonlinear.

2. [24/6] Same as #1 for \( \frac{d^3y}{dt^3} + t \frac{dy}{dt} + (\cos^2 t) y = t^3. \)

3. [25/10] Verify that \( y_1 (t) = \frac{1}{3} t \) and \( y_2 (t) = \frac{1}{3} t + e^{-t} \) are solutions of the 4th order constant coefficient linear differential equation \( y^{(4)} + 4y''' + 3y = t. \)

4. [25/12] Verify that \( y_1 (t) = t^{-2} \) and \( y_2 (t) = t^{-2} \ln t \) are solutions of the 2nd order variable coefficient linear differential equation \( t^2y'' + 5ty' + 4y = 0. \)

5. [25/18] Determine values of the constant \( r \) for which the differential equation \( y''' - 3y'' + 2y' = 0 \) has solutions of the form \( y = e^{rt}. \)

6. [25/20] Determine values of the constant \( r \) for which the differential equation \( t^2y'' - 4ty' + 4y = 0 \) has solutions of the form \( y = t^r \) for \( t > 0. \)

NOTES