Maple Lab, Week 29

Far-Out Integrals II

Based on suggestions by M. L. Platt, CASE Newsletter #28, April 1997

References: Handout for Lab 21 (Far-Out Integrals I);
Improper integrals: Stewart Sec. 7.9;
Infinite series: Stewart Chap. 10

Theme: We continue our investigation of \( I \equiv \int_{0}^{\infty} \frac{\sin x}{x} \, dx \).

Exercises: 5. How do we know that the improper integral \( I \) is convergent? Study the Comparison Theorem for Integrals, Stewart pp. 493–494. Although Stewart never gets around to saying so, the series theorem on p. 635 has an analogue for improper integrals:

If \( \int_{a}^{\infty} |f(x)| \, dx \) converges, then \( \int_{a}^{\infty} f(x) \, dx \) converges.

Therefore, the condition \( f(x) \geq g(x) \geq 0 \) in the comparison theorem can be replaced by \( f(x) \geq |g(x)| \).

(A) Can you apply the comparison theorem directly to the formula above for \( I \)? (Why not?)

(B) Integrate by parts to get an integral to which the comparison theorem applies. Hint:

\[
\int_{0}^{\infty} f(x) \, dx = \int_{0}^{\pi} f(x) \, dx + \int_{\pi}^{\infty} f(x) \, dx.
\]

6. Define \( a_k = \int_{k\pi}^{(k+1)\pi} \frac{\sin x}{x} \, dx \), and consider \( S = \sum_{k=0}^{\infty} a_k \).

(A) Prove that the series \( S \) converges, and that the sum equals \( I \).

(B) Write a Maple procedure to evaluate \( a_k \) numerically for any given \( k \geq 1 \). (The procedure may call \texttt{simpson} from the \texttt{student} package.)

(C) Evaluate \( a_0 \) separately. (You probably already did this if you finished the extra credit Exercise 4 in Part I.)

(D) Add up enough terms in the series \( S \) to approximate \( I \) to 2 decimal places. Hint: Alternating Series Estimation Theorem, p. 632.
7 (extra credit). Get a better approximation to $I$ with less computation, using the sequence

$$b_k = \int_{k\pi}^{(k+1)\pi} \frac{\cos x}{x^2} \, dx.$$

8 (extra credit). Is the series $S$ absolutely convergent? After deciding, make your answer to 5(A) more precise.