(* Discussion *)

(* This demonstration deals with a function of the form $B(x - t)$. We verify that it solves the two-variable wave equation,

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}.$$ 

Then we plot $B(x - t)$ as a function of $x$ for a sequence of values of $t$ and animate the results, verifying that the solution represents a traveling wave. *)

(* Demonstration *)

(* Define the basic function. *)

(* We choose an example that has a localized peak and approaches 0 at the sides, since that will best demonstrate the traveling-wave behavior. (Generally speaking, we use lower case for Mathematica variable names, to avoid possible conflicts with built-in Mathematica function names. *)

\[ b[z_] := \frac{1}{1 + z^2} \]

(* Verify the differential equation. *)

(* Calculate a second derivative. *)

\[ D[b[x-t], \{x,2}\]  
\% - D[b[x-t], \{t,2}\]

(* *)
(* Plot and animate. *)

(* First try a single plot. *)
Plot[b[x-1], {x,-10,10}]

(* Animation is hardware-dependent. *)

(* [[ On a Sun-3 running SunView:

<<Graphics`Animation`
Animate[Plot[b[x-t], {x,-10,10}, PlotRange->{0, 1.2}],
   {t,-12,12,2}]

The plots will briefly appear individually, then the animation will start. ]] *)

(* [[ On a NeXT: *)
Do[Plot[b[x-t], {x,-10,10}, PlotRange->{0, 1.2}],
   {t,-12,12,2}]

(* This will display all the plots sequentially. Use the mouse to mark the large bracket (at right of the window) surrounding the entire list of plots. Then click on the menu item “graphs” and the submenu item “animate selected graphics”. Use the buttons at the bottom of the animation display window to slow down the animation for clarity. ]] *)

(* end *)
Introducing D’Alembert’s solution of the wave equation

These commands are primarily intended to be typed in as a demonstration, rather than input as a file.