1. Let \( f(x) = \sqrt{1 + x^2} \) and let \( c \) be a fixed (but arbitrary) real number. Compute \( f'(c) \) using the definition of the derivative.

2. Suppose that \( f \) is a differentiable function. Evaluate each of the following limits:

\[
\lim_{h \to 0} \frac{f(a + 2h) - f(a)}{h} \quad \text{and} \quad \lim_{h \to 0} \frac{f(a + h) - f(a - h)}{h}.
\]

3. Suppose that \( F, G, \) and \( H \) are differentiable functions. Show that

\[
(FGH)' = F'GH + FG'H + FGH'.
\]

(Begin by putting \( FG = U \); then use the product rule twice.)

4. Let \( n \) be a fixed (but arbitrary) positive integer, and define \( f(x) := x^n|x| \).

(i) Sketch the graph of \( y = f(x) \) for \( n = 1, 2, 3 \).

(ii) Use the definition of the derivative to show that \( f'(0) = 0 \).

(iii) Find \( f'(c) \) for \( c \neq 0 \). (Consider two cases: \( c > 0 \) and \( c < 0 \).)

(iv) Use your findings above to give a general formula for \( f'(c) \), \( c \) a real number.

5. Where is the function \( g(x) = |x - 1| + |x - 2| \) differentiable? Give a formula for \( g' \) and sketch the graphs of \( g \) and \( g' \).

6. For what values of \( a \) and \( b \) is the line \( 2x + y = b \) tangent to the parabola \( y = ax^2 \) when \( x = 2 \)?

7. (taken from Basic Analysis: Japanese Grade 11) Two curves \( y = x^3 + ax \) and \( y = x^2 + bx + c \) pass through the point \((1, 2)\) and have a common tangent line at this point. Find the values of the constants \( a, b, \) and \( c \).

8. Suppose that \( A \) is a fixed positive number.

(a) Find an equation for each of the two tangent lines to the curve \( y = x^2 \) which pass through the point \((0, -A^2)\).

(b) Find the value of \( A \) for which the aforesaid tangents are perpendicular to each other.

9. Let \( C \) denote the graph of the parabola \( y = 1 - x^2 \). Suppose that \( a \) is a (fixed) positive number, and let \( A \) denote the point \((0, 1 + a)\) on the \( y \)-axis. Suppose that the two tangent lines to \( C \) which pass through \( A \) touch \( C \) at the points \( P \) and \( Q \).

(i) Find the co-ordinates of \( P \) and \( Q \) (in terms of \( a \)).

(ii) Find the value of \( a \) such that the triangle formed by \( A, P, \) and \( Q \) is equilateral.
10. Suppose that \( f \) is differentiable at 0,

\[
\lim_{x \to 0} \frac{f(x)}{x} = 4, \quad \text{and} \quad \lim_{x \to 0} \frac{g(x)}{x} = 2.
\]

(i) Find \( f(0) \).
(ii) Find \( f'(0) \).
(iii) Find \( \lim_{x \to 0} g(x) f(x) \).

11. Suppose that \( f \) is a function that satisfies the equation

\[
f(x + y) = f(x) + f(y) + x^2 y + xy^2
\]

for every pair of real numbers \( x \) and \( y \). Assume further that \( \lim_{x \to 0} \frac{f(x)}{x} = 1 \). Find (i) \( f(0) \), (ii) \( f'(0) \), and (iii) \( f'(x) \).