1. Let
   \[ f(x) := \begin{cases} x^2 \sin \left(1/x\right), & \text{if } x \neq 0; \\ 0, & \text{if } x = 0. \end{cases} \]
   (i) Recall from lecture that \( f'(0) = 0 \).
   (ii) Find \( f'(x) \) for every \( x \neq 0 \).
   (iii) What is \( \lim_{x \to 0} f'(x) \)?
   (iv) Explain how the finding in (iii) can be reconciled with (i).

2. Find all points on the curve \( y = (\cos x)/(2 + \sin x) \) at which the tangent is horizontal.

3. Suppose that \( f \) is a differentiable function, and \( b \) and \( m \) are constants. Define \( g(x) = f(b + mx) + f(b - mx) \). Find \( g'(x) \).

4. Suppose that \( f \) is a differentiable function and \( g(t) = [f(sin t)]^2 \). Find \( g'(t) \).

5. Suppose that \( n \) is a fixed positive integer. Show that the following identities hold:
   \[
   \frac{d}{dx} \left[ \sin^n(x) \cos(nx) \right] = n \sin^{n-1}(x) \cos((n+1)x) \\
   \frac{d}{dx} \left[ \cos^n(x) \cos(nx) \right] = -n \cos^{n-1}(x) \sin((n+1)x)
   \]

6. (from an old 151 (common) exam) Suppose that \( f \) is a differentiable function. It is known that the curve \( y = f(x) \) has exactly one horizontal tangent, corresponding to \( x = 2 \). Define \( g(x) = f(x^2 + x) \). Find all values of \( x \) for which the graph of \( g \) has horizontal tangents.

7. Suppose that \( f \) is a differentiable function such that \( f(1) = 1, f(2) = 2, f'(1) = 1, f'(2) = 2, \) and \( f'(3) = 3 \). If \( g(x) = f(x^3 + f(x^2 + f(x))) \), find \( g'(x) \) and hence \( g'(1) \).