Sections 1.1-1.3: Introduction to Differential Equations

Examples:

Verify that $y = Ce^{at}$ is a solution to the ODE $\dfrac{dy}{dt} = a * y$. If we also have the initial condition $y(0) = 10$, what else can we say about the solution? (NOTE: a differential equation with an initial condition is called an **initial value problem**)

Find the values of r for which $y = t^r$ is a solution to the ODE $t^2 y''' - 4ty'' + 4y' = 0$.

Suppose an object is dropped from rest. What forces act on it? Use Newton's second law to create an initial value problem for the velocity of the object. Classify this equation as done in the previous example, and show that $y = \dfrac{mg}{c} - \dfrac{mg}{c} e^{-ct/m}$ is a solution to the IVP.

Given a differential equation of the form $\dfrac{dy}{dt} = f(t, y)$, we can visualize the families of solutions using a **direction field**:

$$y' = \frac{1}{e^t - y}$$

$$y' = -2y$$