

## 2.5: Autonomous Differential Equations and Population Models

### Examples:

Identify and classify the equilibrium points of  $y' = y^2 - 4y + 3$  Also determine the intervals of increase/decrease and concavity of  $y$  as discussed in the Prep Assignment.

Solve the ODE to confirm your answers above. This is an example of a **logistic** population growth model.

Another type of growth model is a **logistic threshold** model:

$$y' = -r \left(1 - \frac{y}{T}\right) \left(1 - \frac{y}{K}\right) y$$

$r$ : **growth rate of the population**

$K$ : **maximum sustainable population**

$T$ : **population threshold (minimum needed for survival)**

Use technology to plot the direction field if  $r = 0.1$ ,  $T = 1000$ , and  $K = 10000$

Find and classify the equilibrium points for general  $r$ ,  $T$ , and  $K$  assuming  $r > 0$ .

Describe the long-term behavior of the population if  $0 < y(0) < T$  and if  $y(0) > T$ .