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In [1]: from sympy import *
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Example 1: Find the fundamental set of solutions to the ODE $y'' + 2y' + y = 0$

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In [2]: t=symbols('t')
y=Function('y')
deq=diff(y(t),t,2)+2*diff(y(t),t)+y(t)
ysoln=dsolve(deq,y(t))
print('The solution to the ODE is',ysoln)
```

The solution to the ODE is Eq(y(t), (C1 + C2*t)*exp(-t))

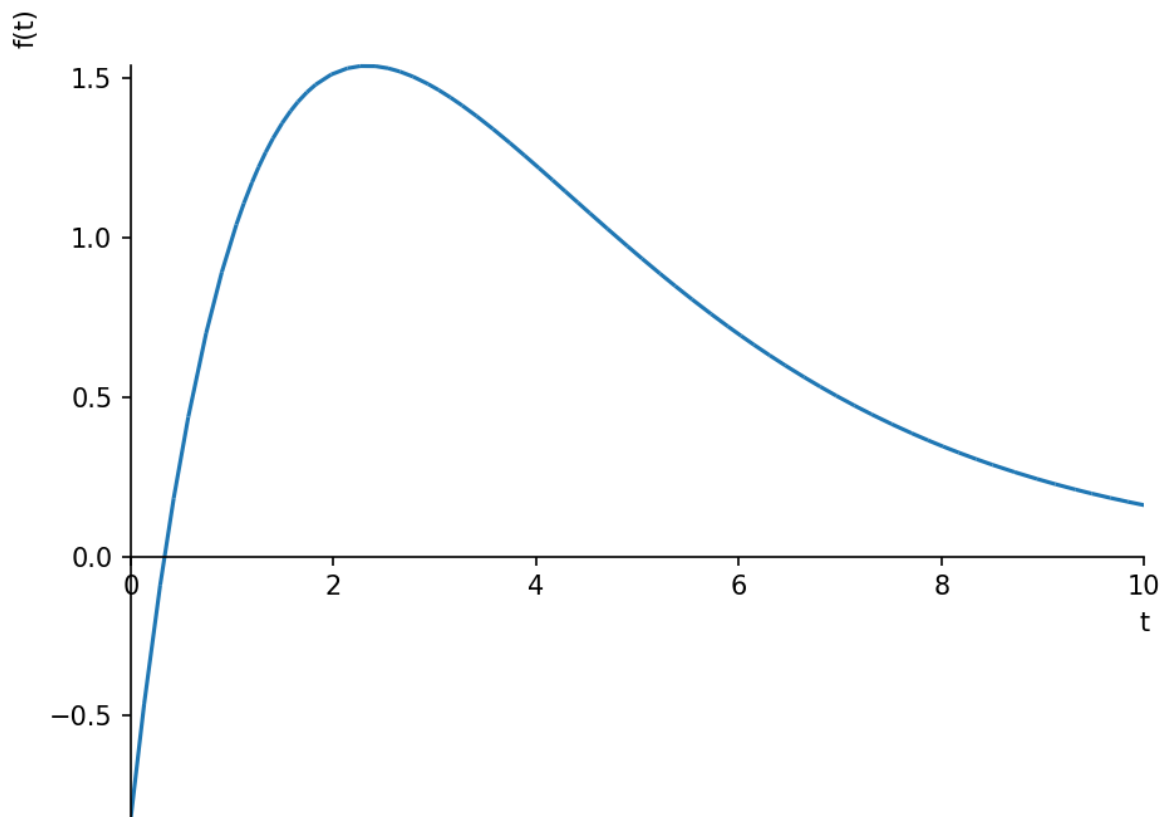
Example 2: Solve the IVP $4y'' + 4y' + y = 0$, $y(1)=1$, $y'(1)=1$. Plot the solution and describe its behavior as $t \rightarrow \infty$

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In [3]: t=symbols('t')
y=Function('y')
deq=4*diff(y(t),t,2)+4*diff(y(t),t)+y(t)
ysoln=dsolve(deq,y(t),ics={y(1):1,diff(y(t),t).subs(t,1):1})
print('The solution is',ysoln)
```

The solution is Eq(y(t), (3*t*exp(1/2)/2 - exp(1/2)/2)/sqrt(exp(t)))

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In [4]: matplotlib notebook
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In [6]: yplot=ysoln.rhs
plot(yplot,(t,0,10))
print('The function approaches 0 because of the e^(-1/2t).')
```



The function approaches 0 because of the $e^{(-1/2t)}$.

$t^2 y'' + 2t y' - 2y = 0, t > 0$ Verify y_1 and y_2 form a Fundamental Set of solutions

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In [9]: t=symbols('t')
y=Function('y')
deq=t**2*diff(y(t),t,2)+2*t*diff(y(t),t)-2*y(t)
ysoln=dsolve(deq,y(t))
print('The solution to the ODE is',ysoln)
# Use Wronskian to show Fundamental Set
y1=t**(-2)
y2=t
Wronsk=wronskian([y1,y2],t)
print('The Wronskian is',Wronsk)
is0=solve(Wronsk)
print('The Wronskian is zero when t=',is0,'so the solutions form a Fundamental Set.')
```

The solution to the ODE is $\text{Eq}(y(t), C_1/t^{**2} + C_2*t)$

The Wronskian is $3/t^{**2}$

The Wronskian is zero when $t= []$ so the solutions form a Fundamental Set.

In []: