In [1]:
```
from sympy import *
```

Example 1: y'' + y' - 6y = 0, y(0) = 1, y'(0) = 2

(Showing using **dsolve** first, then demonstrating the use of Laplace transforms)

Strategy: enter the Laplace Transform of the LHS by hand, then apply the Laplace Transform on the RHS.

In [3]:
```
t=symbols('t')
y=Function('y')
deq=diff(y(t),t,2)+diff(y(t),t)-6*y(t)
ysoln=dsolve(deq,ics={y(0):1,diff(y(t),t).subs(t,0):2})
print('Solution to the IVP is',ysoln)
s,t=symbols('s t',positive=True)
Y=Function('Y')
y0=1
yp0=2
lap_deq=s**2*Y(s)-s*y0-yp0+s*Y(s)-y0-6*Y(s) # RHS transform is 0
print(' ')
print('The Laplace transform of the ODE is',lap_deq)
Yofs=solve(lap_deq,Y(s))
print(Yofs) # to see how many solutions produced.  Since only one, it is th
e ZEROth solution***
print('Solving for Y(s) yields',Yofs[0])
ysoln=inverse_laplace_transform(Yofs[0],s,t)
print('y=',ysoln)
```

```
Solution to the IVP is Eq(y(t), exp(2*t))

The Laplace transform of the ODE is s**2*Y(s) + s*Y(s) - s - 6*Y(s) - 3
[1/(s - 2)]
Solving for Y(s) yields 1/(s - 2)
y= exp(2*t)
```

Example 2: y'' + y = sin(2t), y(0)=2, y'(0)=1

In [5]:
```
t=symbols('t')
y=Function('y')
deq=diff(y(t),t,2)+y(t)-sin(2*t)
ysoln=dsolve(deq,y(t),ics={y(0):2,diff(y(t),t).subs(t,0):1})
print('Direct solution of the ODE is',ysoln)
s,t=symbols('s t',positive=True)
Y=Function('Y')
y0=2
yp0=1
Lg=laplace_transform(sin(2*t),t,s)
print(' ')
print('The Laplace transform of the RHS is',Lg)
print('Note that we want the first (zeroth) value: second is if s<0')
lap_deq=s**2*Y(s)-s*y0-yp0+Y(s)-Lg[0]
print('The laplace transform of the ODE is',lap_deq)
Yofs=solve(lap_deq,Y(s))[0]  # again only one solution
print('Solving for Y(s) yields',Yofs,'(NOTE Python combines all into one fr
action)')
ysoln=inverse_laplace_transform(Yofs,s,t)
print('y=',ysoln)
```

```
Direct solution of the ODE is Eq(y(t), 5*sin(t)/3 - sin(2*t)/3 + 2*cos(t))

The Laplace transform of the RHS is (2/(s**2 + 4), -oo, True)
Note that we want the first (zeroth) value: second is if s<0
The laplace transform of the ODE is s**2*Y(s) - 2*s + Y(s) - 1 - 2/(s**2 +
4)
Solving for Y(s) yields (2*s**3 + s**2 + 8*s + 6)/(s**4 + 5*s**2 + 4) (NOTE
Python combines all into one fraction)
y= 5*sin(t)/3 - sin(2*t)/3 + 2*cos(t)
```

Example 3: y'' + 2y' + y = 4e^(-t), y(0)=2, y'(0)=-1

In [7]:
```
t=symbols('t')
y=Function('y')
deq=diff(y(t),t,2)+2*diff(y(t),t)+y(t)-4*exp(-t)
ysoln=dsolve(deq,y(t),ics={y(0):2,diff(y(t),t).subs(t,0):-1})
print('Direct solution of the ODE is',ysoln)
s,t=symbols('s t',positive=True)
Y=Function('Y')
y0=2
yp0=-1
Lg=laplace_transform(4*exp(-t),t,s)
print(' ')
print('The Laplace transform of the RHS is',Lg,'(want zeroth element agai
n)')
lap_deq=s**2*Y(s)-s*y0-yp0+2*s*Y(s)-2*y0+Y(s)-Lg[0]
print('The Laplce transform of the IVP is',lap_deq)
Yofs=solve(lap_deq,Y(s))[0]
print('Solving for Y(s) yields',Yofs,'(again Python combines into one fract
ion)')
ysoln=inverse_laplace_transform(Yofs,s,t)
print('y=',ysoln)
```

```
Direct solution of the ODE is Eq(y(t), (2*t**2 + t + 2)*exp(-t))

The Laplace transform of the RHS is (4/(s + 1), -oo, True) (want zeroth ele
ment again)
The Laplce transform of the IVP is s**2*Y(s) + 2*s*Y(s) - 2*s + Y(s) - 3 -
4/(s + 1)
Solving for Y(s) yields (2*s**2 + 5*s + 7)/(s**3 + 3*s**2 + 3*s + 1) (again
Python combines into one fraction)
y= (2*t**2 + t + 2)*exp(-t)
```

In [ ]: