A proof of Prop. 3.8(c)

(by S.A.F. with a little bit of help from the Instructor's Manual)

By definition of "interior", B and D are on the same side of $\overleftrightarrow{AC}$, which is the same as $\overleftrightarrow{AE}$. We must show that also B and E are on the same side of $\overleftrightarrow{AD}$. If they are not, then segment BE intersects either $\overrightarrow{AD}$ or $\overrightarrow{AF}$ at a point $G \in \overrightarrow{BE}$. Clearly $G \neq A$, since A, B, and C are not collinear. By Exercise 9, therefore, G and E are on the same side of $\overleftrightarrow{AB}$, and G and B are on the same side of $\overleftrightarrow{AE}$.

Case 1: $G \in \overrightarrow{AD}$. By Exercise 9 again, G and D are on the same side of $\overleftrightarrow{AB}$. So E and D are on the same side of that line by B-4. But D and C are on the same side, by the other half of the definition of interior of $\angle CAB$. So E and C are on the same side, which contradicts the construction of E.

Case 2: $G \in \overrightarrow{AF}$. By Exercise 9 once more, G and F are on the same side of $\overleftrightarrow{AC}$. In the proof of (b) we saw that F and B are on opposite sides of $\overleftrightarrow{AC}$. Therefore, B and G are on opposite sides of $\overleftrightarrow{AC}$. This contradicts our previous conclusion that B and G are on the same side of $\overleftrightarrow{AE}$.