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Team Lambda

Proposition 3.8: If D is in the interior of <CAB, then

(a) so is every other point on ray $\overrightarrow{AD}$ except A;

(b) no point on the opposite ray to $\overrightarrow{AD}$ is in the interior of <CAB; and

(c) if C*A*E, then B is in the interior of <DAE.

Proof of (a): Let there be a point $P \in \overrightarrow{AD}$. Then A*P*D or A*D*P (Exercise 9). In both cases, P is on the same side of $\overleftrightarrow{AC}$ as D and also the same side of $\overleftrightarrow{AB}$ as D. Then by definition of *interior*, P is in the interior of <CAB.

Proof of (b): Let $\overrightarrow{AF}$ be a ray opposite to ray $\overrightarrow{AD}$; that is, D*A*F, where F lies on $\overleftrightarrow{AD}$ and $\overrightarrow{AD} \cup \overrightarrow{AF} = \overleftrightarrow{AD}$ and $\overrightarrow{AD} \cap \overrightarrow{AF} = \{A\}$ by Exercise 9. Let there be a point $P \in \overrightarrow{AF}$, so P*A*D. Assume P is in the interior of <CAB (RAA). Then P is on the same side of $\overleftrightarrow{AC}$ as B and also on the same side of $\overleftrightarrow{AB}$ as C. Then by definition of *interior*, P would be on the interior of <CAB. However, a contradiction occurs because P and D are on opposite sides of $\overleftrightarrow{AC}$, so P and B are on opposite sides of $\overleftrightarrow{AC}$. Therefore, P is not in the interior of <CAB.

Proof of (c): We are given $\overrightarrow{AD} \cup \overrightarrow{AF} = \overleftrightarrow{AD}$, $\overrightarrow{AD} \cap \overrightarrow{AF} = \{A\}$, F*A*D, $B \notin \overleftrightarrow{AC}$ (because Greenberg's definition of angle does not include 180° angles, or straight angles. <CAB is understood to be <180°).

By part (a), every other point in $\overrightarrow{AD}$ except A lies on the interior of <CAB. Therefore, C and B are on opposite sides of $\overleftrightarrow{AD}$. Since C*A*E, E lies on $\overleftrightarrow{AC}$ (and $\overleftrightarrow{AC} = \overleftrightarrow{AE}$) and E and B lie on the same side of $\overrightarrow{AD}$. Therefore, because <CAB is <180° and E lies on $\overleftrightarrow{AC}$, E cannot lie on the interior of <CAB. Therefore, B and D must lies on the same side of $\overleftrightarrow{AC}$. Therefore, because B lies on the same side of $\overleftrightarrow{AC}$ as D and B lies on the same side of $\overleftrightarrow{AD}$ as E, B must be in the interior of <DAE.

QED.