Group work: MU [with corrections by SAF]
October 6, 2010.

Q: 12 page 148

Prove proposition 3.9 (Hint: For proposition 3.9 (a), use Pasch's theorem and proposition 3.7; see figure 3.37. For proposition 3.9 (b) let the ray emanate from point D in the interior of triangle ABC. Use the crossbar theorem and proposition 3.7 to show that ray AD meets BC in a point E such that A*D*E. Apply Pasch's theorem to Triangle ABE and triangle AEC: see figure 3.38.)

Proposition 3.9:

a) If a ray r emanating from an exterior point of triangle ABC intersects side AB in a point between A and B, then r also intersects side AC or side BC.
b) If a ray emanates from an interior point of triangle ABC, then it intersects one of the sides, and if it does not pass through a vertex it intersects only one side.

Prop. 3.9 (a)

Proof:

1. Pasch's theorem guarantees that the line containing the ray r (that is, the line XD in the figure) intersects either AC or BC. (see figure 3.37 in the text, but note that X could be on the far side of the triangle from D).
2. Assume, for example, that the line XD intersects AC at point E
3. We want to show that E is in the ray XD; that is, ¬E*X*D (B-3 and definition of a ray).
4. Since every point between D and E is interior to triangle ABC (Prop. 3.7) but X is not, then, as we were hoping for, ¬ E * X *D (definition of exterior of triangle).
5. Therefore, r intersects side AC.

qed

Prop. 3.9(b)

Proof:

1. Let the ray emanate from point D in the interior of triangle ABC. (see figure 3.38 in the text)
2. By the crossbar theorem ray AD intersects BC at a point, say E.
3. By Proposition 3.7 we have A * D * E.
4. By Pasch's theorem applied to triangle ABE and ACE the line through D containing the given ray r will meet either AB or BE at F and either AC or CE at G.
5. Proposition 3.7 ensures that F * D * G, so the given ray contains exactly one of F and G.

qed