Math. 467 (Fulling) 16 October 2015

Midterm Test – Solutions

Name: ______________________________

1. *(Multiple choice – each 5 pts.) (Circle the correct capital letter.)*

(a) Suppose A $*$ B $*$ C and A $*$ C $*$ D. Which of these is *not* true?

(A) C $*$ B $*$ A
(B) D $*$ B $*$ A
(C) B $*$ C $*$ A
(D) B $*$ C $*$ D
(E) D $*$ C $*$ A

C

(b) Which of the following is *not* provable from the Hilbert I and B axioms?

(A) If D is in the interior of $\angle$CAB, then so is every other point on $\overrightarrow{\mathrm{AD}}$ except A.

(B) Every point D in the interior of $\angle$CAB lies on a segment joining a point E on $\overrightarrow{\mathrm{AB}}$ to a point F on $\overrightarrow{\mathrm{AC}}$.

(C) If D lies on line $\overleftrightarrow{\mathrm{BC}}$, then D is in the interior of $\angle$CAB if B $*$ D $*$ C.

(D) If $\overrightarrow{\mathrm{AD}}$ is between $\overrightarrow{\mathrm{AC}}$ and $\overrightarrow{\mathrm{AB}}$, then $\overrightarrow{\mathrm{AD}}$ intersects segment BC.

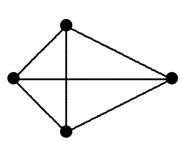
(E) If D lies on line $\overleftrightarrow{\mathrm{BC}}$, then B $*$ D $*$ C if D is in the interior of $\angle$CAB.

B (This is the famous "Warning".)

(c) The Greek approach to geometry was revolutionary because

(A) they discovered the Pythagorean theorem and divided a circle into 360 degrees.
(B) they insisted upon proofs.
(C) they were not interested primarily in practical applications.
(D) they focused upon idealizations such as infinitely thin and infinitely straight lines.
(E) [all of these except (A)]

E

(d)  This diagram is used in the proof of

(A) Pasch's theorem
(B) crossbar theorem
(C) SAS
(D) ASA
(E) SSS

E

(e) There are some differences between Hilbert's axioms and Euclid's. Which of these is *not* one? (That is, the correct item is the same in both systems. The others are true only of Hilbert's.)

(A) SAS is an axiom, not a theorem.
(B) A line can be extended indefinitely.
(C) "Betweenness" is an explicit and central concept.
(D) Circle-cirle continuity is no longer tacitly assumed.
(E) Congruence of all right angles is a theorem, not an axiom.

B (This is part of both Euclid II and Hilbert C-1.)

(f) In geometry as formulated in Greenberg's book, "angles" are confined to which range of angular measure?

(A) $0^\circ < \theta < 180^\circ$
(B) $-180^\circ < \theta \leq 180^\circ$
(C) $0^\circ \leq \theta < 180^\circ$
(D) $0^\circ < \theta < 360^\circ$
(A) $0^\circ \leq \theta \leq 180^\circ$

A

---

2. *(15 pts.)* Prove the *segment subtraction theorem*: If A $*$ B $*$ C, and D $*$ E $*$ F, and AB $\cong$ DE, and AC $\cong$ DF, then BC $\cong$ EF.

[See Exercise 20, p. 150.]

3. *(10 pts.)* Simplify $\neg\forall x\, \exists m\, [x \geq 0 \land S(m,x) \;\Rightarrow\; \forall n\, Q(m,n,x)]$.
(Push the "$\neg$" in as far as you can!) (In Greenberg's notation, $\neg$ is $\sim$, and $\land$ is &.)

$$\exists x\, \forall m\, \neg[x \geq 0 \land S(m,x) \;\Rightarrow\; \forall n\, Q(m,n,x)].$$

$$\exists x\, \forall m\, \neg[\neg(x \geq 0 \land S(m,x)) \lor \forall n\, Q(m,n,x)].$$

$$\exists x\, \forall m\, [(x \geq 0 \land S(m,x)) \land \neg\forall n\, Q(m,n,x)].$$

$$\exists x\, \forall m\, [x \geq 0 \land S(m,x) \land \exists n\, \neg Q(m,n,x)].$$

4. *(25 pts.)* Recall that Axiom B-4 states:

For any line l and any three points A, B, C not lying on l,

(i) If A and B are on the same side of l and if B and C are on the same side of l, then A and C are on the same side of l.

(ii) If A and B are on opposite sides of l and if B and C are on opposite sides of l, then A and C are on the same side of l.

(a) Prove the corollary,

(iii) If A and B are on opposite sides of l and if B and C are on the same side of l, then A and C are on opposite sides of l.

By definition, points are on opposite sides if and only if they are not on the same side (assuming, always, that none of the points is *on* the line). So we should be able to prove (iii) by assuming A and C are on the same side and deriving a contradiction. Make that assumption and reletter (i) this way: If A and C are on the same side of l and if C and B are on the same side of l, then A and B are on the same side of l. Thus the RAA hypothesis and the second hypothesis of (iii) contradict the first hypothesis of (iii).

(b) Explain how this axiom defines a "side" and shows that a line has exactly two sides.

(i) shows that "being on the same side" is transitive and therefore is an equivalence relation. It partitions the points that don't lie on l into equivalence classes, which we can call "sides". Then (ii) shows that there are only two sides (all points that are not in the same side as B are on the same side as each other).

5. *(Essay – 20 pts.)* Explain what the *Euclidean parallelism property* is, and what it means to say that the Euclidean parallel postulate is *independent* of other axioms. Stress the importance of *models*, and illustrate these ideas in the context of finite incidence geometries (sets of finitely many points and lines that satisfy the three "I" axioms).