Group Alpha Exercise 6

Uniqueness 4.3

 Let M be the midpoint of AB. Then A\*M\*B. Let M’ also be the midpoint of AB. Then A\*M’\*B. By 3.3, we see that either A\*M\*M’\*B or A\*M’\*M\*B (or M=M’). If M does not equal M’, then AM is not equal to AM.’ By trichotomy:

Case 1: AM<AM’. Then MB>MB’. But AM=MB by definition of midpoint. So then AM>MB’. But MB’=AM’. So then AM>AM’. This is a contradiction, thus this case is impossible.

Case 2: (Works similarly) AM>AM’. Then MB<MB’. But MB=AM. So then AM<MB’. But MB’=AM’. This gives AM<AM’ which is also a contradiction, thus this case doesn’t work as well.

Thus M=M’, which proves that the midpoint of a segment is unique.

Proof 4.4

 Suppose you want to bisect an angle ABC. Without loss of generality, assume ABC creates an isosceles triangle (can simply select an E on ray AB st. BC=AE). Let D be the midpoint of the side opposite of the angle being bisected (AC) in this case, st A\*D\*C. Thus AD=CD. Now connect points B and D. Then by SSS, we know these two triangles are congruent, triangle ABD=triangle CBD. This means each correspond angle is congruent. Thus angle ABD=angle CBD. Thus segment BD is the angle bisector of this angle, (by angle addition) since A\*D\*C and D is between the angle ABC. The bisector is unique since any bisector of this angle will also be on this segment, thus the bisector is unique.