Math. 467 (Fulling) 3 March 2017

Midterm Test – Solutions

Name: ______________________________

1. *(Multiple choice – each 5 pts.) (Circle the correct capital letter.)*

   (a) Which of the following is *not* provable from the Hilbert I and B axioms?

   (A) If $\overrightarrow{AD}$ is between $\overrightarrow{AC}$ and $\overrightarrow{AB}$, then $\overrightarrow{AD}$ intersects segment BC.

   (B) If D lies on line $\overleftrightarrow{BC}$, then D is in the interior of $\angle$CAB if B * D * C.

   (C) If D lies on line $\overleftrightarrow{BC}$, then B * D * C if D is in the interior of $\angle$CAB.

   (D) If D is in the interior of $\angle$CAB, then D lies on some segment joining a point E on $\overrightarrow{AB}$ to a point F on $\overrightarrow{AC}$.

   (E) If D is in the interior of $\angle$CAB, then so is every other point on $\overrightarrow{AD}$ except A.

D (This is the notorious "Warning" on p. 115. The others are all propositions on pp. 115–116.)

   (b) Let P stand for the proposition "All right angles are congruent" and let Q stand for the SAS triangle congruence criterion. Which of the following is true?

   (A) Both P and Q were proved as theorems by Euclid.

   (B) Both P and Q were assumed as axioms by Euclid.

   (C) Q can easily be proved from P.

   (D) Modern geometry has P as an axiom and Q as a theorem, but in Euclid's writing it was the reverse.

   (E) Modern geometry has Q as an axiom and P as a theorem, but in Euclid's writing it was the reverse.

E (See p. 122 and p. 128.)

   (c) The simplest example of a *hyperbolic* incidence geometry has

   (A) 3 points.

   (B) 4 points.

   (C) 5 points.

   (D) 7 points.

   (E) [none of these]

C

   (d) In geometry as formulated in Greenberg's book, "angles" are confined to which range of angular measure?

   (A) $0^\circ < \theta < 180^\circ$

   (B) $-180^\circ < \theta \leq 180^\circ$

   (C) $0^\circ \leq \theta < 180^\circ$

   (D) $0^\circ < \theta < 360^\circ$

   (E) $0^\circ \leq \theta \leq 180^\circ$

A

---

2. *(15 pts.)* State the three **congruence** axioms that don't involve angles. (These are the first three.)

[See pp. 119–120.]

3. *(15 pts.)* Recall that Axiom B-4 states:

   For any line l and any three points A, B, C not lying on l,

   (i) If A and B are on the same side of l and if B and C are on the same side of l, then A and C are on the same side of l.

   (ii) If A and B are on opposite sides of l and if B and C are on opposite sides of l, then A and C are on the same side of l.

   Explain how this axiom defines a "side" and shows that a line has exactly two sides.

[Make the following points in a good essay: (1) (i) says that "same side" is transitive. (2) It is reflexive and symmetric, so it's an equivalence relation. (3) The "sides" are the cells of the resulting partition. (4) (ii) shows (how?) that there are only 2 sides.]

4. *(10 pts.)* Simplify $\neg\exists x\,\forall y\,\big[\big(x \le 0 \vee S(x,y)\big) \wedge \exists n\,\big(T(y,n) \wedge n > x\big)\big]$.
   (Push the "$\neg$" in as far as you can!) (In Greenberg's notation, $\neg$ is $\sim$, and $\wedge$ is &.)

$$\forall x \exists y\,\neg\,\big[\big(x \le 0 \vee S(x,y)\big) \wedge \exists n\,\big(T(y,n) \wedge n > x\big)\big]$$

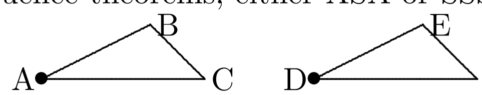
$$\forall x \exists y\,\big[\neg\big(x \le 0 \vee S(x,y)\big) \vee \neg\exists n\,\big(T(y,n) \wedge n > x\big)\big]$$

$$\forall x \exists y\,\big[\big(x > 0 \wedge \neg S(x,y)\big) \vee \forall n\,\neg\big(T(y,n) \wedge n > x\big)\big]$$

$$\forall x \exists y\,\big[\big(x > 0 \wedge \neg S(x,y)\big) \vee \forall n\big(\neg T(y,n) \vee n \le x\big)\big]$$

5. *(20 pts.)* State and prove **ONE** of the triangle congruence theorems, either ASA or SSS.

   (Extra credit for doing both is limited to 10 points.)



6. *(Essay – 20 pts.) IMPROVEMENTS IN THE REWRITE GET ONLY HALF CREDIT. E.G., IF YOUR INITIAL SCORE IS 16, YOUR FINAL SCORE WON'T EXCEED 18.*

   Proposition 2.4 is, "For every point, there is at least one line not passing through it." One of your teammates has proposed the following proof:

   > According to Axiom I-3, there are three points (call them A, B, and C) such that no line is incident with all of them. Let P be A. Then P does not lie on line $\overleftrightarrow{BC}$, QED.

   Explain what is wrong with this proof. Then give a correct proof.