Math. 467 (Fulling) 3 May 2019

Final Examination – Solutions

Name: ______________________________

1. *(Multiple choice – each 5 pts.)*

   (a) In hyperbolic geometry
   (A) the "AAA" triangle congruence criterion is valid.
   (B) the circumference of a circle is less than 2π times the radius.
   (C) the angles of a triangle add to $180°$.
   (D) parallel lines are equidistant.

A

   (b) The arc-length formula $ds^2 = (dx^2 + dy^2)/y^2$ characterizes the
   (A) Klein disk.
   (B) Poincaré disk.
   (C) Poincaré half-plane.
   (D) Beltrami pseudosphere.

C

   (c) Which of these statements is **FALSE** in hyperbolic geometry?
   (A) Every valid theorem of neutral geometry remains valid.
   (B) Rectangles do not exist.
   (C) Summit angles of Saccheri quadrilaterals are acute.
   (D) Two parallel lines always have a common perpendicular.

D

   (d) In what respect is Poincaré's disk model better than Klein's?
   (A) It is *conformal*, meaning that it represents angles accurately.
   (B) It satisfies a stronger continuity axiom.
   (C) Its lines are straight Euclidean lines.
   (D) Its segment lengths are ordinary Euclidean lengths.

A

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2. *(15 pts.)* Explain this apparently paradoxical statement;

   If you succeed in proving the parallel postulate, you will have shown that Euclidean geometry is **inconsistent**.

See Greenberg, pp. 289–293.

3. *(30 pts.)* Rearrange these names into historical order, earliest to latest:

   Hilbert, Hypatia, Euclid, Lobachevsky, Thales, Saccheri

Thales, Euclid, Hypatia, Saccheri, Lobachevsky, Hilbert

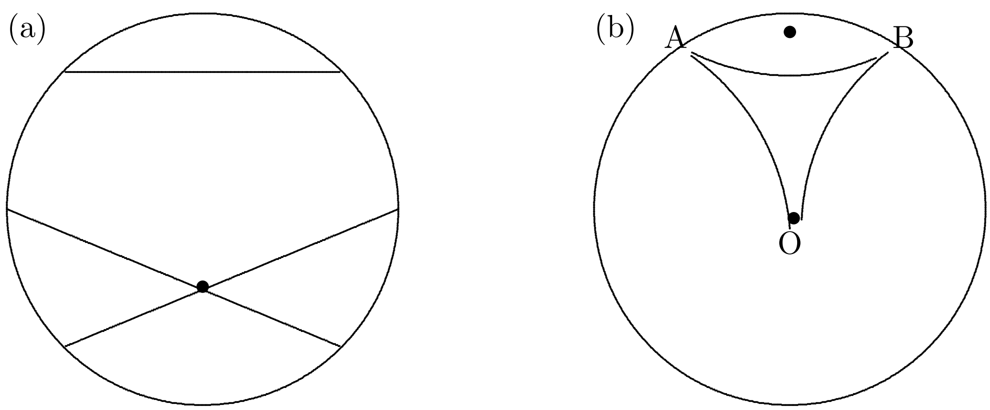
4. *(30 pts.)* Recall **Wallis's Axiom**:

   Given any triangle ΔABC and any segment DE, there exists a triangle ΔDEF having DE as one of its sides such that ΔABC ∼ ΔDEF (that is, ∠A ≅ ∠D, ∠B ≅ ∠E, ∠C ≅ ∠F).

   Prove that Wallis's axiom is **equivalent** to the Euclidean parallel postulate (either HE or EV). *Hint:* In one direction, you can use the fact that the angles of a triangle sum to 180°.

See Greenberg, pp. 216–217 and p. 229. Note that HE is known to imply the triangle-sum formula, but we don't know (to start with) whether Wallis does.

5. *(10 pts.)*
   (a) In the Klein disk, draw a sketch showing how the parallel postulate (HE) can be violated.
   (b) In the Poincaré disk, draw a sketch showing how a point can be inside an angle but not connected to the sides of the angle by any line.



A)(B is the limiting parallel ray to the angle sides. Cf. Fig. 7.9 of Greenberg. The arcs should be perpendicular to the circle.

6. *(20 pts.)* Prove **ONE** of the famous triangle congruence theorems, ASA or SSS. **If you try both, clearly indicate which one you want graded.**

See Greenberg, p. 151–152, for ASA. For SSS, recall homeork and class discussion. In both cases, recall that these are neutral geometry theorems from Chapter 3 (they would have been on the midterm exam if it had been a week later). Therefore, you should not be using later theorems from Chapter 4 (such as hypotenuse-leg) and certainly can't use "angle sum = 180 degrees" since that requires the parallel postulate.

7. *(Essay – 25 pts.)*

State the Alternate Interior Angle Theorem **and** its converse, and discuss their significance. (In particular: Tell which of them can be proved from the Hilbert IBC axioms and which cannot. Explain which of them excludes elliptic geometry and which excludes hyperbolic geometry.)