Definition of the factor space:

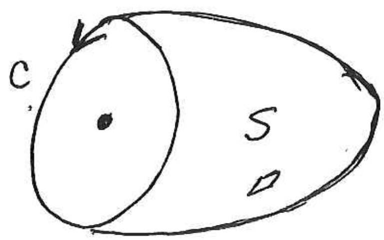
$H^1(\Omega) \equiv$ (irrotational)/(gradients)

$H^2(\Omega) \equiv$ (solenoidal)/(curls)

"Dangerous" holes:

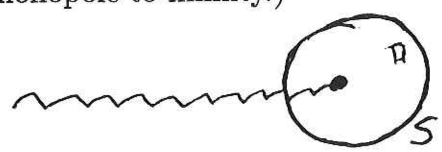
are those that can be lassoed (e.g., infinitely extended line singularities). (Consider the magnetic field due to an electrical current in a wire. It is not possible to define a "magnetostatic scalar potential" all around the wire.)

Note that *point* singularities are now no obstacle to showing $\oint \vec{A} \cdot d\vec{x} = 0$.



are those that can be caught in sacks (possibly nonspherical). Examples:

A) point singularity. (If a magnetic monopole exists, its magnetic field can't be derived from a vector potential ($\vec{W}$) defined everywhere in $\mathbf{R}^3 - \{\vec{0}\}$; somewhere there must be a "Dirac string" connecting the monopole to infinity.)



B) ring singularity surrounding line singularity (can be put into a toroidal sack). (A ring of magnetic monopoles has a magnetic field not derivable from a vector potential.)

Periods:

are line integrals around elementary singularities. Irrotational $\vec{A} \Rightarrow$ period is independent of details of curve by Stokes's theorem:

$$\oint_{C_1 - C_1'} \vec{A} \cdot d\vec{x} = \int_S (\nabla \times \vec{A}) \cdot d\vec{S} = 0.$$

are surface integrals around elementary singularities. Solenoidal $\vec{A} \Rightarrow$ period is independent of details of surface by Gauss's theorem:

$$\oint_{S_1 - S_1'} \vec{A} \cdot d\vec{S} = \int_V (\nabla \cdot \vec{A})\, d^3\vec{x} = 0.$$